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ABSTRACT

Results from the first year of a three-year longitudinal study (grades 6-8) of sex-related differences are reported. The purposes of the study are: (1) to understand more adequately the impact of one important cognitive influence (Spatial Visualization) on the learning of mathematics, and (2) to identify educational influences on the development of feelings of confidence in the learning of mathematics. Influences on confidence are being studied by observing classroom interactions and by a projective-type interview. Subjects in this portion of the study are above the mean in achievement and either high or low in confidence in learning mathematics. The impact of spatial visualization skills on the learning of mathematics is being studied in a one-to-one interview with subjects who are discrepant in spatial/verbal skills. They are asked to solve pictorial or analytic word and fraction problems. Transcribed interviews are coded for verbal information, picture information, picture use, mental movement, and scores. (Author/MK)

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Symposium

Cognitive and Affective Influences on the Developments of Sex-Related Differences in Mathematics

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That women are under-represented in occupations related to mathematics is an assertion that needs no support. Hypotheses as to why this occurs range from those concerned with genetic differences between the sexes which limit females' ability to learn mathematics, to overt discrimination which limits females' employment opportunities. However, such hypotheses are not particularly helpful in understanding why women are under-represented in occupations related to mathematics or in increasing the representation. Even assuming some validity for the genetic hypothesis (which many doubt, [Nelson, 1977; Sherman, 1977]), the number of women in careers related to mathematics is much less than can be explained by the kinds of genetic differences researchers hypothesize. Blaming overt discrimination for women's under-representation does not help in understanding why adult females have less knowledge and fewer skills in mathematics than do adult males (as clearly indicated by the First National Assessment of Educational Progress results [Mullis, 1975]). Until adult females' mathematical knowledge and skills are equivalent to those of adult males, unequal representation of the sexes in occupations related to mathematics will continue. Therefore, the major problem facing those concerned with equity in this area

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is ensuring that females acquire mathematical skills and knowledge equivalent to males. In order to do this, knowledge about why females do not learn mathematics is essential.

Although simplistic to state, this is not a simple problem. Involved in it is the cognitive acquisition of mathematics by females, as well as the attitudes or affective beliefs held by females, male peers, parents, and educators toward females as learners of mathematics. The cognitive and affective components are so intertwined that it is difficult, if not impossible, to separate them. Not only are they intertwined, but they are developed over a period of years in a complex social matrix which involves home, community, and school. Therefore, in seeking to understand why inequity exists in the representation of the sexes in occupations related to mathematics, one must study the cognitive and affective components affecting the acquisition of mathematical skills and knowledge in the social environment where such skills are developed.

Although intellectually pleasing to discuss, at the present time it is impossible to study the totality of causative behavior. However, it is possible, and indeed profitable, to select subelements of major influence from each important component and to study the development, interrelationships and effects of these subelements upon the learning of mathematics. This is the focus of the study described here. The most important cognitive and affective variables related to the differential learning of mathematics by females and males are being studied in depth over a period of three years. Specifically, the problem addressed is: 1) To determine the influence of spatial visualization skills and feelings of confidence upon sex-related differences in mathematics, 2) To ascertain the development and stability of these influences by studying the same

students over a three year period, and 3) To study these influences in the environment which most affects mathematics learning, i.e, the school.

In order to achieve the purposes of the study, a longitudinal design was necessary. Three years was chosen as the maximum feasible length of study. Four schools in a midwest city which include grades 6-8 are participating in the study. What we are reporting here are the results of the first year data collection and analyses. All subjects were in the sixth grade when data were collected. The conclusions reached are all tentative. In many cases we feel the analyses are incomplete but decided that it would be of value to report our first year findings.

THE AFFECTIVE COMPONENT

In a recent review of the literature about affective variables (or attitudes) and mathematics learning, Fennema (1977) concluded that "The confidence/anxiety dimension as it relates to mathematics learning is one of the more important affective variables that helps explain sex-related differences in mathematics learning (p. 32, 40)."

In the Fennema-Sherman study (Fennema & Sherman, 1978), at each grade level from 6 to 12, males were significantly more confident in their abilities to deal with mathematics than were females. In most instances, this result was found when there were no significant sex-related differences in mathematics achievement. In addition, confidence in learning mathematics and mathematics achievement were more highly correlated than any other affective variable and achievement ($r \simeq .40$). What is unknown at the present time is how these feelings of confidence/anxiety develop, their stability over a period of time, and what influences their development.

What influences females to be less confident than males in mathematics even when they apparently achieve as well? Do many males and teachers send subtle and not so subtle messages that mathematics is not feminine? Do females sense a declining power in themselves as problem-solvers? Do females, more than males, attribute causation of successes and failures in mathematics in such a way to make them doubtful of future success? Several specific factors which have been hypothesized as important influences on the development of confidence were investigated.

Attribution of Success and Failure in Mathematics

Attributional style appears to be related to feelings of self-esteem or confidence, which in turn is an important influence on achievement. The

attributional model that appears to be the most useful to educators as an aid in understanding academic achievement is the one proposed by Weiner (1974). In this model, attributions of success and failure are categorized into the matrix shown in Figure 1, with locus of control being one dimension and stability the other.

Attributions of past successes and failures to certain of the Weiner categories have been shown to be related both to task persistence and task choice (Bar-Tal, 1978). If one attributes success to internal causes, particularly the internal stable cause, ability, then one can expect success in the future and will be more apt to approach or persist at certain tasks. If, however, success is attributed to external factors, success in the future is not assured and one will avoid the task. A somewhat different situation is true of failure attributions. If one attributes failure to unstable causes, failure can be avoided in the future. Thus, the tendency to approach or persist at tasks will be encouraged. Attribution of failure to a stable cause, on the other hand, will lead one to believe that failure cannot be avoided.

Although one should be extremely careful of overgeneralizing data and concluding that all males behave one way and all females another way, many studies have reported that females and males tend to exhibit different attributional patterns (Deaux, 1976; Bar-Tal & Frieze, 1977). In a somewhat simplistic summary, males tend to attribute successes to internal causes and failures to external or unstable causes. Females tend to attribute successes to external or unstable causes and failures to internal causes.

	Internal	External
Stable	Ability	Task
Unstable	Effort	Environment

Causes of Success or Failure

Figure 1

This particular combination of attributions (success attributed externally and failures attributed internally) has been hypothesized to strongly affect academic achievement and in particular, females' achievement. Bar-Tal (1978, p. 267) states that "females and individuals with certain causal perceptions may perform in a classroom below their abilities because of their maladaptive patterns of attributions."

One facet of the relationship between sex-related differences in attribution and achievement has to do with the stereotyping of the activity involved. Stereotyping of task behavior as male or female does have an influence on how causal attributions are made (Deaux, 1976). The differences between the sexes in attributions is lessened, but not eliminated, when the task under consideration is considered appropriate for the sex under consideration. E.g., when subjects are asked why a male succeeds on a masculine task, attributions tend to be in the ability category. When the same question is asked about a female succeeding in a masculine task, attributions tend to fit into the effort category. When subjects are asked about either male or female success in a female task, the causation of male success is still more apt to fit the ability category than are the causations attributed to the female's success, but the differences are lessened. Differences are minimized but not reversed when the task is stereotyped female (Deaux, 1976).

A recent study using mathematics success and failure experiences as the focus confirmed that similar sex-related differences in attribution exist in the domain specific area of mathematics (Wolleat, Pedro, Becker & Fennema, 1979). The influence of attributional style was investigated in the study reported here.

Participation in High Level and Low Level Cognitive Activities

There is evidence that males perform better than females on higher level cognitive (process) tasks and that females perform better than males on lower level cognitive (product) tasks (Fennema, 1980). High level cognitive tasks become increasingly important for success in mathematics classes as learners progress through high school. If females begin to sense a lack of ability to perform high level cognitive tasks, they may indicate this by lowered feelings of confidence. There are few sex-related differences in mathematics achievement when amount of mathematics studied is controlled. Most research reinforces that finding. Achievement tests, however, do not measure high level cognitive tasks very well. Many females have informally reported they sensed a lack of ability to do more abstract mathematics or high level cognitive tasks. If there are indeed sex differences in high level cognitive tasks, this is a potent explanation of sex-related differences in mathematics occupations. Such a hypothesis is currently receiving some support from the Second National Assessment of Educational Progress (Fennema & Carpenter, Note 1).

One check on the existence of such cognitive level differences and also a possible explanation of such differences, is time spent on high or low cognitive tasks and teacher-student interaction at differing cognitive levels. Pupils learn what they do, thus if more time is spent on high level cognitive tasks, students will learn to do such tasks. Pupils also learn what is expected of them. If teachers and peers expect certain pupils, i.e., males, to perform well on high level cognitive tasks and have different expectations for females, differential learning will result. Both of these hypotheses were investigated.

Teacher/Peer Influences

While no one doubts that the social milieu outside of schools affects females' and males' willingness to learn mathematics, it is within the school itself that mathematics is learned. The social milieu of the school appears to be the most important determinant of students' attitudes toward mathematics and their feelings about themselves as learners of mathematics. After three years of intensive study of sex-related differences in mathematics, Fennema and Sherman (1978) note "the possibility must be considered that a major source of determinants of sex-related differences in mathematics learning is located within schools themselves. . . .Further research is needed into factors within a school milieu that might contribute to the negative effects on girls in relation to mathematics learning" (p. 201, 202). The most important area within schools where attitudes toward mathematics are formed is the classroom. Classrooms are composed of learners and teachers interacting in a vareity of ways. In a comprehensive review of classroom research, Brophy and Good (1974) conclude that the patterns of interaction between teachers and students differ greatly depending on the sex and achievement level of the students. Teachers interact with male students more than with female students and also direct more attention to high achieving students than to low achieving students. Brophy & Good (1974), however, find that female and male mathematics teachers do not differ significantly in their treatment of female and male students.

THE CONFIDENCE STUDY

The methodologies utilized in studying the influences on confidence and the sample studied are represented in Figure 2. The sample was first identified by giving an achievement test to all sixth graders in four middle schools. The confidence level of all students was measured by the

Fennema-Sherman Confidence in Mathematics Scale. Girls and boys above the mean in mathematics achievement and who scored above the 75% percentile and below the 25% percentile in confidence were selected as the sample.

Each subject in the sample was observed in her/his mathematics classrooms and also participated in a one-one interview. Both the observations and the interview investigated attributional style, process-product participation, expectancies by teachers and peers of boys and girls. Each data collection procedure investigated these components in somewhat different ways.

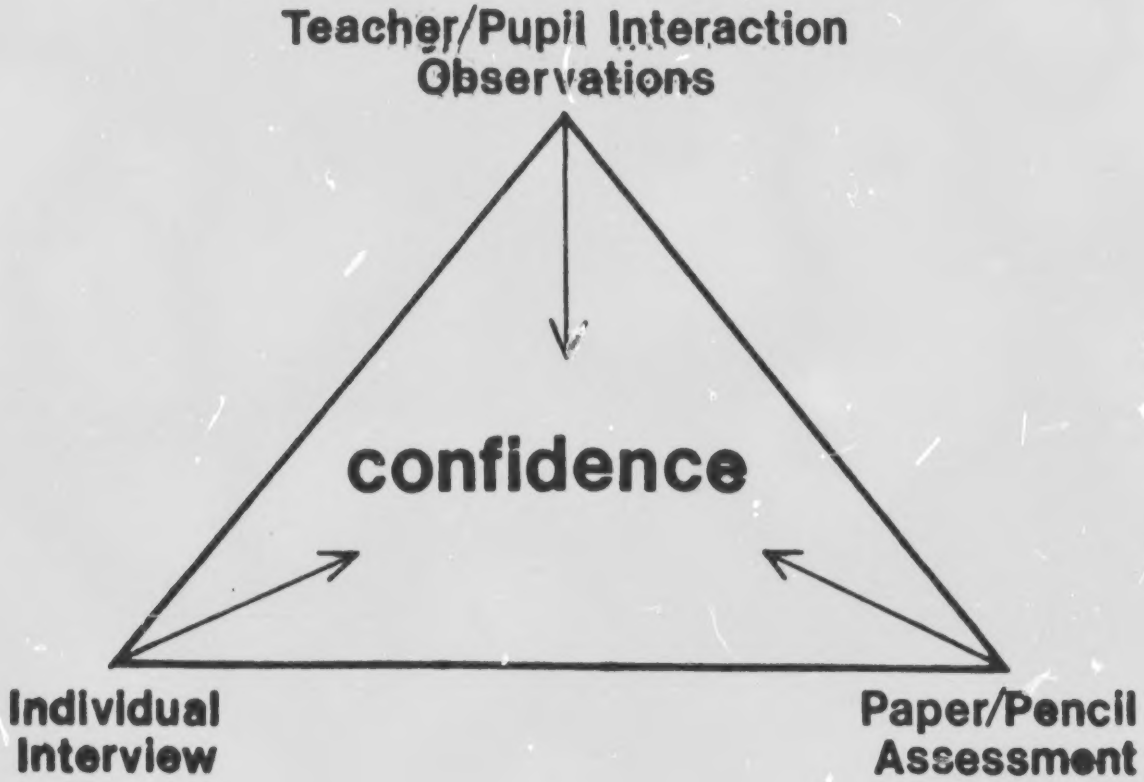
TEACHER-STUDENT INTERACTIONS

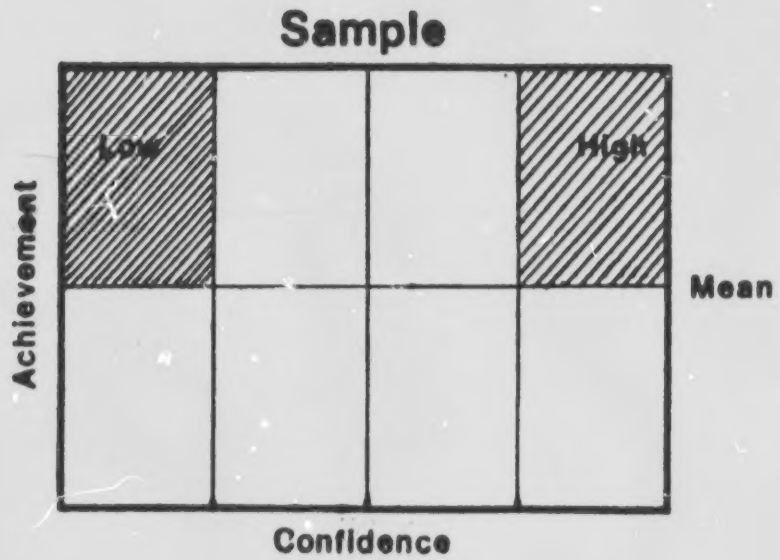
Since it appears the classroom environment is important in the development of sex-related differences in mathematics achievement, this study focused on interactions between individual mathematics teachers and individual students. The objective of the study was to identify any differences in the patterns of interaction between teachers and their female and male students who had been identified as high or low in their confidence in learning mathematics.

There is a large body of literature on teacher-student interaction research. In a review of that literature, Brophy and Good (1974) wrote:

> Many student attributes have been found to influence teachers' perceptions of students and their patterns of interaction with them in the classroom. Some of these include student membership in various identifiable groups (social class, race, sex) while others are individual differences in personal attributes or classroom behavior and achievement. (Brophy & Good, 1974, p. 6)

THE AFFECTIVE COMPONENT





Females and Males

Figure 2

One of the student attributes that affects teacher-student interactions is the sex of the student (Brophy & Good, 1974). There is considerable evidence that teachers interact more with male students than with female students. This is true both for interactions initiated by the teacher and interactions initiated by the student. Boys generally receive more criticism for their behavior than do girls, and also boys often receive more praise and positive feedback than do girls. Boys seem to be more salient in the teacher's view than girls.

One study (Good, Sikes, & Brophy, 1973) using seventh- and eighth-grade classrooms, found that teachers gave high achieving boys most favorable attention, followed by high achieving girls, low achieving girls and low achieving boys. The authors concluded from this data that "male students per se, in secondary schools, are not being treated inappropriately by teachers of either sex" (Good, Sikes & Brophy, 1973, p. 85). They make no mention of inequity in the fact that high achieving boys received more favorable treatment from their teachers than did high achieving girls.

Dweck and her colleagues (Dweck, Davidson, Nelson, & Enna, 1978; Dweck & Bush, 1976) have done a series of studies on sex differences in learned helplessness in achievement situations. Learned helplessness exists when failure is seen as insurmountable and is attributed to uncontrollable factors such as lack of ability. Her results emphasize the importance of teachers, particularly in their feedback for failure. In a study of teacher-student interactions at the fifth-grade level, 54.4% of work-related criticism to boys referred to intellectual inadequacy while 88.9% of work-related criticism to girls was addressed to intellectual performance (Dweck et al., 1978). In conclusion, Dweck et al. state "that the pattern of evaluative feedback given to boys and girls in the classroom

can result directly in girls' greater tendency to view failure feedback as indicative of their level of ability" (Dweck et al., 1978, p. 274). Thus, both amount and type of interaction between teachers and female and male students are important.

Another attribute of students that is important in determining the patterns of interaction that occur between teacher and students is student achievement level (Brophy & Good, 1974). In fact, Brophy and Good indicate achievement is "perhaps the most important individual difference" (1974, p. 14) that influences teacher-student interactions. After reviewing studies at both elementary and secondary levels, Brophy and Good conclude that "teachers interact more frequently with high achieving than low achieving students, and that their interactions with high achievers are more positive and facilitative than their interactions with low achievers" (1974, p. 16). For this reason, it is important to take achievement into account when observing students.

In addition, other personal attributes of students affect the interactions that go on between teachers and students (Brophy & Good, 1974). Student characteristics such as physical attractiveness, peer popularity, conformity, independence, assertiveness, and orderliness are related to the quantity and quality of interactions teachers have with students. Students' level of confidence in their ability to learn mathematics has not been studied with respect to its influence on patterns of teacher-student interactions in mathematics classes. Considering those characteristics of students which have been found to affect patterns of interactions, it seems possible that students' confidence may indeed be related to the quantity and types of interactions that occur.

Thus, a study of teacher-student interactions which takes into account

mathematics achievement and student sex as an approach to understanding sex-related differences in mathematics, is a reasonable approach to gaining further understanding of sex-related differences in mathematics.

Data Collection and Analyses

Sixty-four students in eight sixth-grade mathematics classes were observed in the Spring 1979 data collection. These students were selected on the basis of their mathematics achievement, confidence level, and sex as described earlier. Classrooms were selected for observation which had 6-9 eligible students. Eight classrooms were observed. The size for each sample group is shown below.

		Confidence: High	Confidence: Low	
Sex	Female	15	17	32
	Male	17	15	32
		32	32	

Observers marked categories to describe the characteristics of each interaction which occurred between the teacher and an observed student in the mathematics class. Among the characteristics coded were: Total number of interactions, No-interaction days, Public vs. Non-public interactions, Cognitive level of interactions, Mathematics vs. Non-mathematics interactions, Initiator of interactions, Feedback by teachers, and Attributions of student success/failure by teachers.

A great deal of care went into the training of the six observers. Each observer trained for three half days with videotapes of middle school mathematics classes developed specifically for this purpose. Observers also trained for three half days in the schools. This in-school training

consisted of practice using the coding system and discussion of differences in coding among the six observers. Observer agreement (simple percent agreement) was checked with a criterion observer in the live classroom situation. Training continued until each observer was able to code with a minimum of 70% agreement with criterion.

Teachers agreed to allow two observers in their mathematics class every day for the duration of one content unit. The content unit was to last a minimum of 15 days. Neither the teachers nor the students knew which students were being observed. Great care was taken to ensure that no student or teacher saw any material which would disclose the identity of the subjects.

The data for the study were collected during March and April 1979. Two types of observation were done in each of the eight classes, however, time only permits discussion of one, teacher-student interaction. The teacher-student interaction data were collected in a systematic, non-participant, live observation using an adaptation of the Brophy-Good Dyadic Observation System (Brophy & Good, 1970).

The data were analyzed by obtaining frequencies within the different categories. Frequencies were summed across all students in a sample group (e.g., high confidence females), across all teachers, and across all days of observation. Thus, for each category, four frequencies were obtained -- one for each of the four sample groups. The frequencies reported here were adjusted for opportunity to interact by taking into consideration student absences and differences in the number of days observed. The adjustment was made by adding or subtracting a certain percentage of the observed frequency to account for differences in the opportunity to interact. All frequencies reported here are adjusted frequencies.

The frequency data were analyzed by the use of percents and ratios. Two types of percents were obtained. Between groups, percents were used to indicate the way particular behaviors were distributed among: (1) females and males, (2) high confidence and low confidence students, and (3) high confidence females, low confidence females, high confidence males, and low confidence males. For example, for Cognitive Level of Interactions, of all higher order interactions 51% were with females and 49% were with males; 52% were with highs and 48% were with lows; and 18% were with high females, 32% with low females, 34% with high males, and 16% with low males. Percents were also obtained within groups to show how each sample group was involved in the behaviors for a category. For example, of all interactions that high confidence females participated in, 22% were higher order interactions and 78% were lower order interactions. For certain categories within group percents are expressed as ratios. For example, the ratio of higher order to lower order interactions for high confidence females was about 2:8. Then these ratios may be compared between groups.

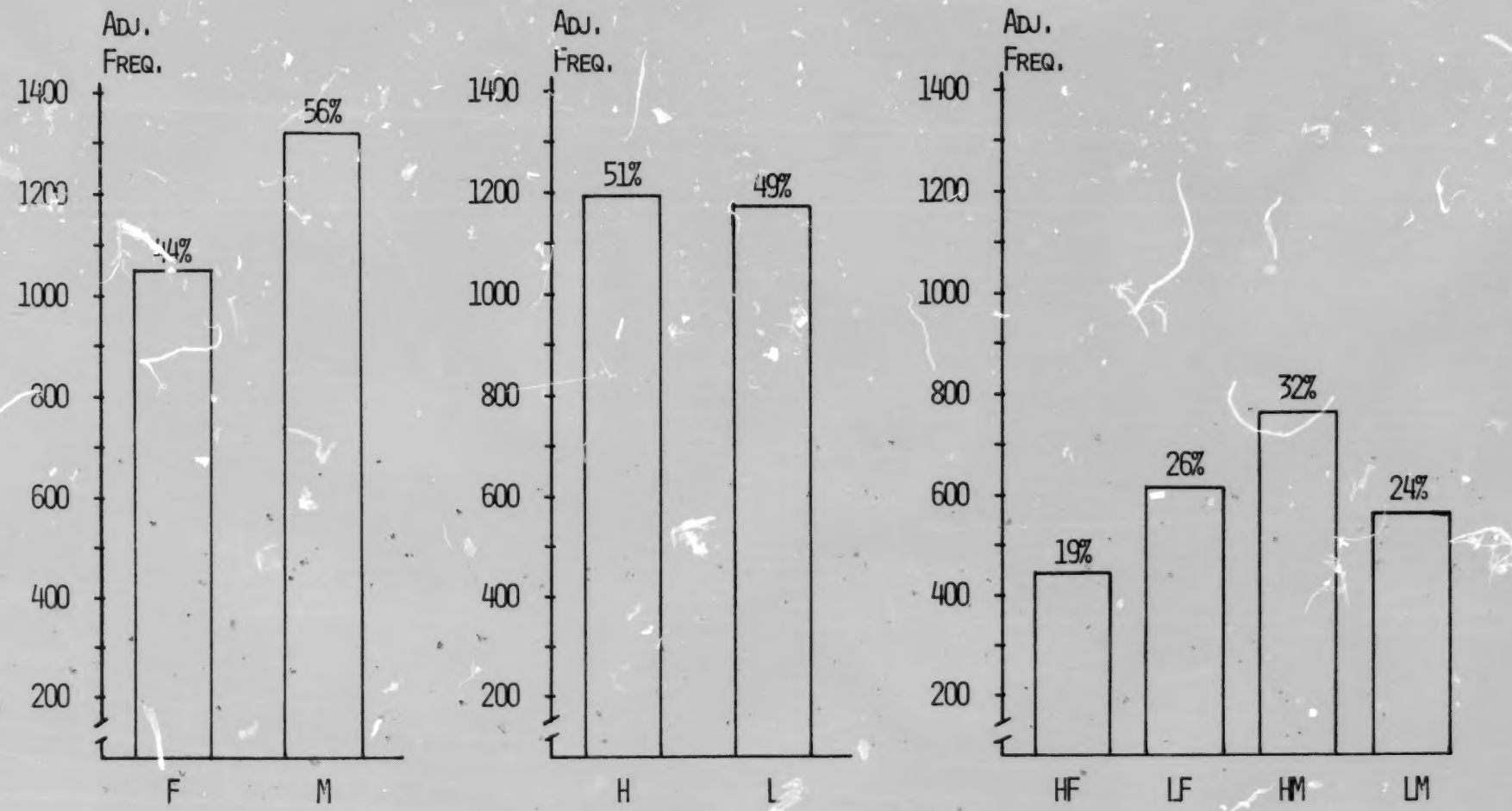
Results

Total Interactions

There were sex differences in the number of interactions between students and teachers (See Figure 3). Males engaged in 56% of all interactions while females engaged in only 44% of all interactions. Students who differed in confidence in their ability to learn mathematics were involved in approximately equal amounts of the Total interactions. Considering both sex and confidence of students, it was the high confidence males who were involved in most interactions (32% of Total interactions) while high confidence females were involved in fewer interactions than the other groups of students (19% of Total interactions). Low confidence males and females were involved in about the same percent of interactions.

Figure 3

Total Interactions



No-Interaction Days

A comparison was made between the number of days when students were observed and the number of those days when students had no interactions with the teacher (Figure 4). Females had more No-interaction days (21% of the time) than did males (17% of the time). Confidence alone was not important in determining which students had No-interaction days. Considering sex and confidence of students together, the high confidence females had No-interaction days more often (24% of time) than any of the other three groups. High confidence males had No-interaction days less frequently (15% of time) than the other three groups.

Public vs. Non-Public Interaction

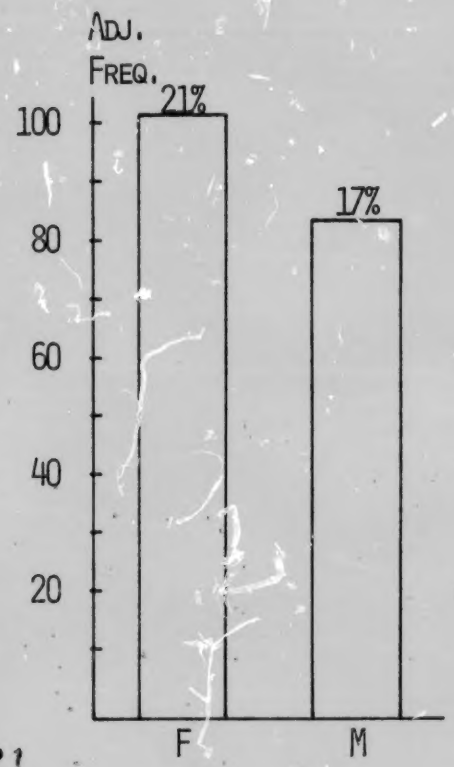
Interactions were coded by whether they occurred in a whole class (Public) or individual (Non-public) setting. Males had more of both Public and Non-public interactions (See Figure 5) than did females, but the ratio of Public to Non-public interactions was about the same for both females and males. High and low confidence students had approximately the same number of interactions, but the ratio of Public to Non-public interactions was 60:40 for high confidence students and 50:50 for low confidence students. High confidence males had more Public interactions (35% of all Public interactions) than the other three groups. High confidence females had fewer Non-public interactions than the other three groups (17% of all Non-public interactions).

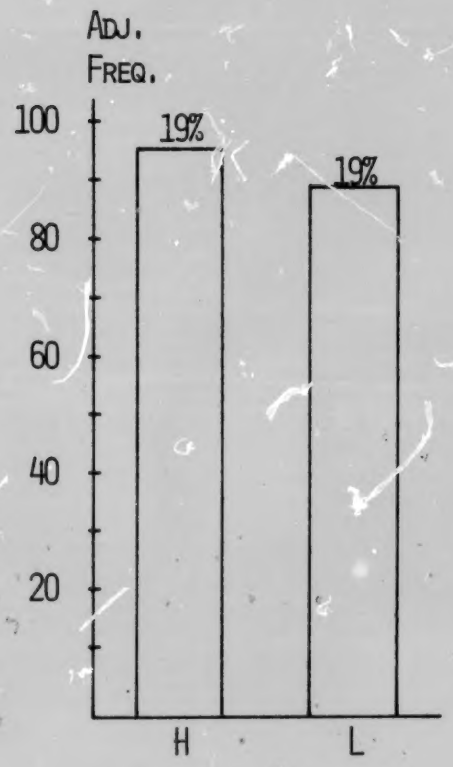
Mathematics vs. Non-Mathematics Interaction

Each interaction was coded as being related to the mathematics content or not related to the mathematics content. The Non-mathematics interactions included behavioral interaction, procedural interactions, and interactions

Figure 4

No Interaction for a Student on a Day





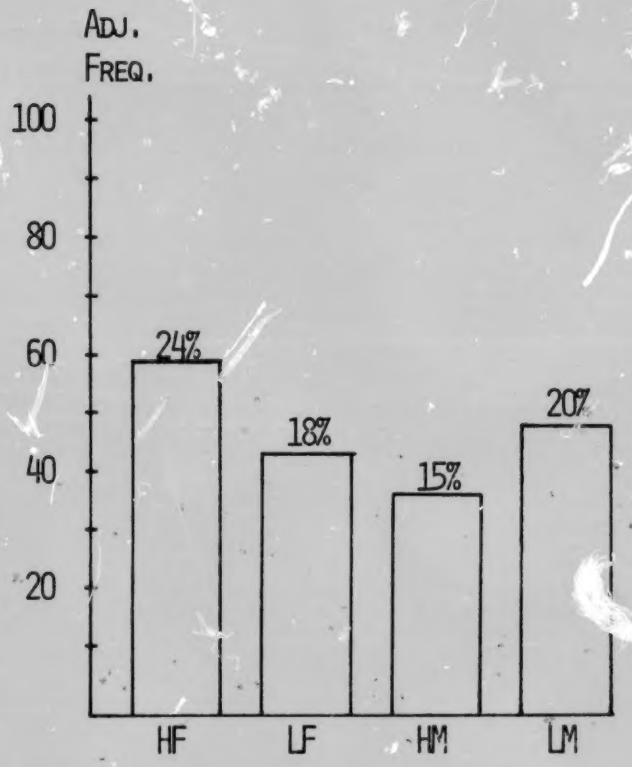


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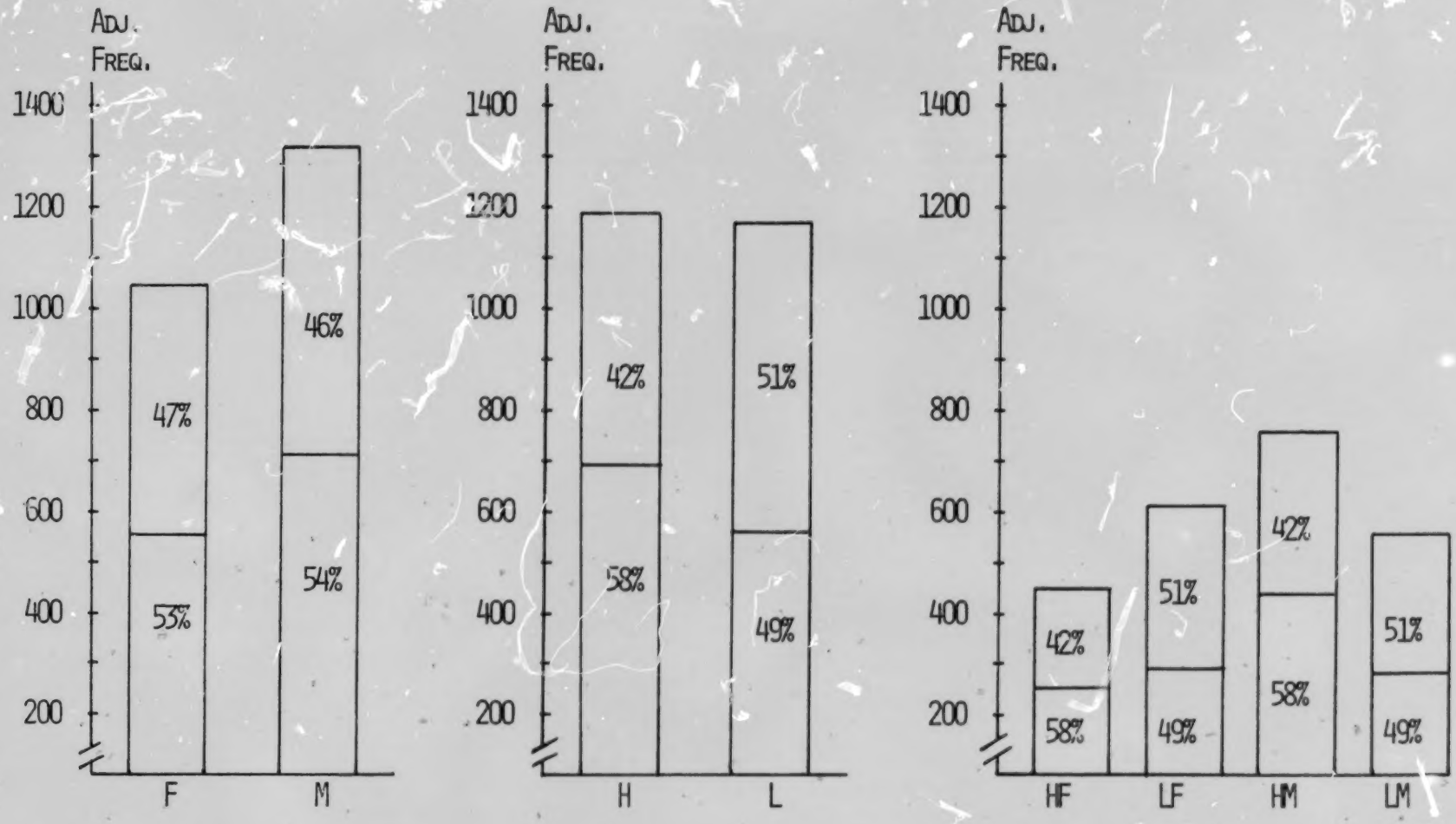
Figure 5

Public vs. Non-Public



unrelated to the content of mathematics class. Females and males were involved in approximately equal percents of Mathematics-related interactions (Figure 6). Students of varying confidence level were also involved in about equal percents of Mathematics interactions. Considering both sex and confidence, those who were involved in most Mathematics interactions were the high confidence males and the low confidence females. Since females were involved in fewer Total interactions than males, it is of interest that females and males were involved in roughly equal number of Mathematics interactions. The interactions which appear to be creating this discrepancy are the Non-mathematics interactions. Males were involved in nearly twice as many Non-mathematics interactions as were females. When considering both sex and confidence of students, most Non-mathematics interactions were with high confidence males and low confidence males (33% and 31%). The group which received fewer Non-mathematics interactions than the other three groups was high confidence females (15%). One may also compare the within group percents for different sample groups. The ratio of Mathematics to Non-mathematics interactions was about 9:2 for high confidence females and low confidence females, but about 5:2 for high confidence and low confidence males.

Cognitive Level of Interaction

Each Mathematics interaction was categorized by cognitive level as being either Higher order or Lower order. Of all Higher order interactions (See Figure 7), about one third were with low confidence females, one third with high confidence males, one sixth with high confidence females, and one sixth with low confidence males. Examining within group data, the ratio of Higher order to Lower order interactions was approximately the same for females and males, and also for high confidence and low confidence students. When both sex and confidence of students

Figure 6

Mathematics vs. Non-Mathematics

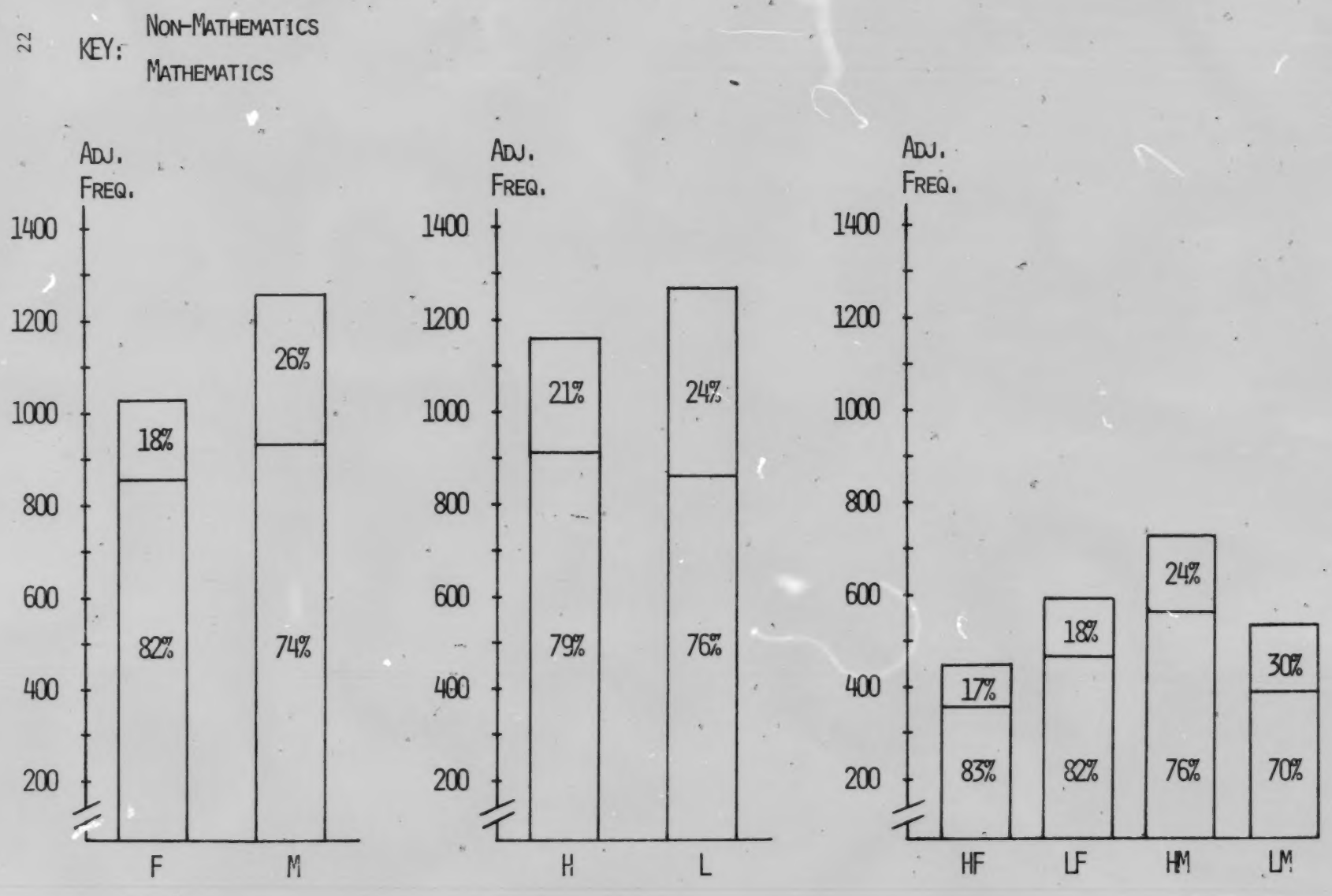
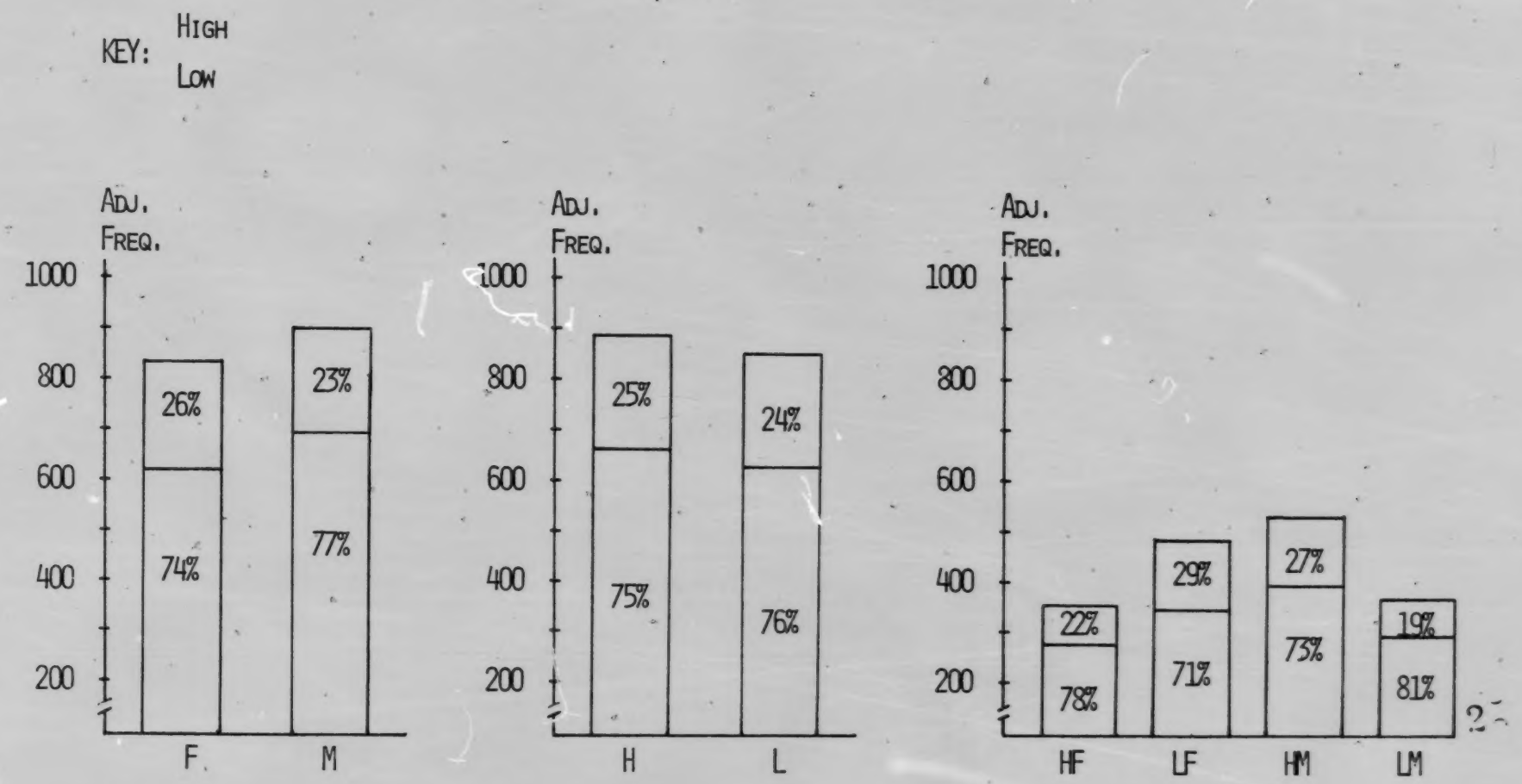


Figure 7

Cognitive Level



were taken into consideration, the ratio of Higher order to Lower order interactions was approximately the same, 2:8 for high confidence females and low confidence males. For low confidence females and high confidence males, the ratio of Higher to Lower order interactions was about 2:5.

Initiator of Interaction

The initiator of each interaction was recorded. Students initiated 37% of all interactions (Figure 8) whereas teachers initiated 63% of interactions. This 4:6 ratio for Student-initiated to Teacher-initiated interactions holds true for females as a group, males as a group, high confidence students, and low confidence students. When considering both sex and confidence of students, only the high confidence females deviated from the ratio of the other three groups. The high confidence females had a Student to Teacher ratio of 3:7 while the other three groups had a ratio of approximately 4:6.

Males initiated more interactions than did females. Fifty-nine percent of all Student-initiated interactions came from males and 41% from females. High and low confidence students initiated about the same number of interactions with their teachers. High confidence males and high confidence females were the extremes for the range of percent of Student-initiated interactions with 34% and 15%, respectively.

Teachers initiated more interactions with males than with females, and also initiated more interactions with high confidence students than with low confidence students. Taking into consideration both sex and confidence of student, the high confidence males were involved in more Teacher-initiated interactions (31%) than the other three groups.

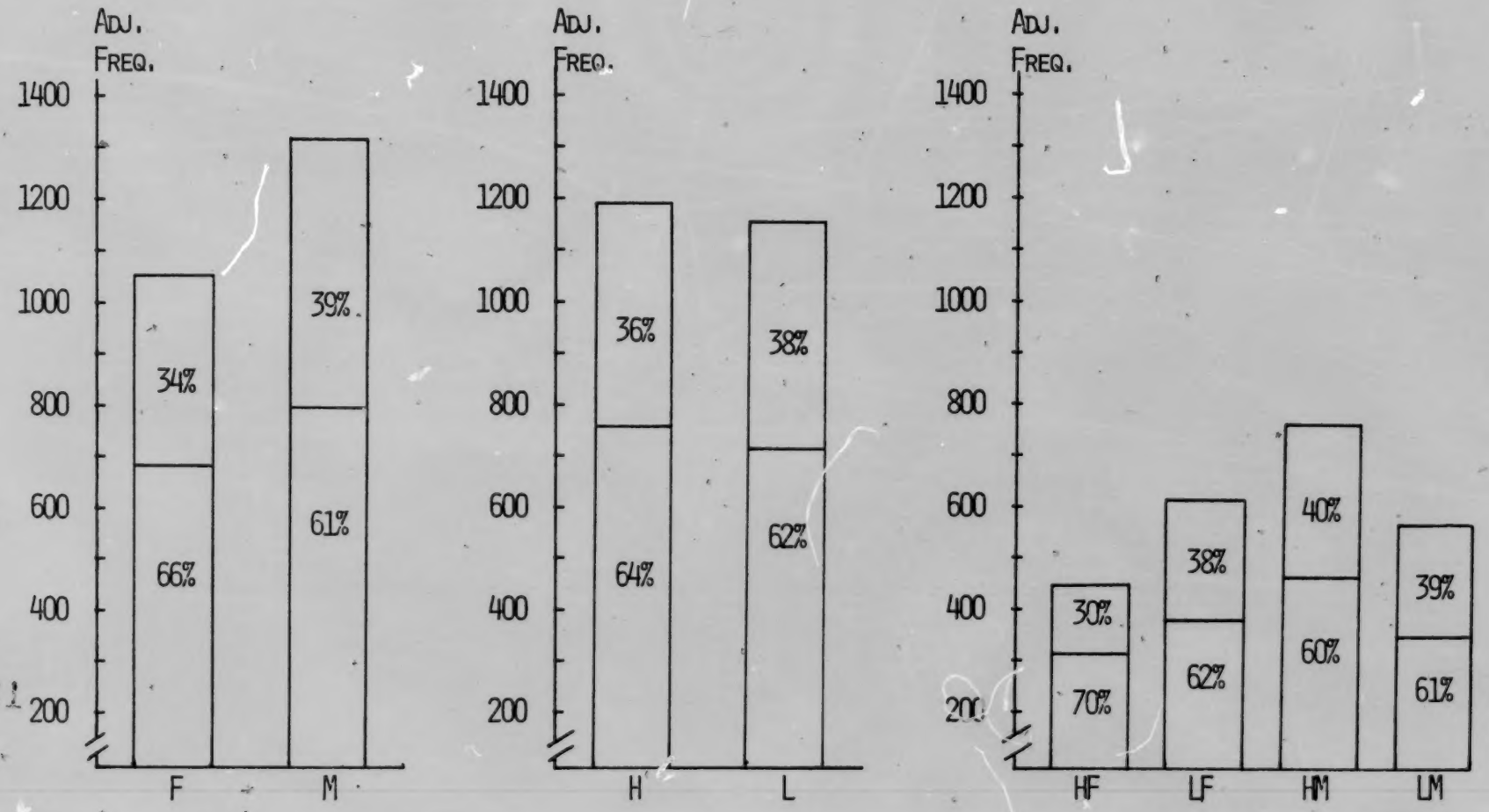
Feedback by Teachers

Feedback to females was about the same as Feedback to males. Feedback

Figure 8

Initiator of Interaction

KEY: STUDENT-INITIATED
TEACHER-INITIEATED



to high confidence students and low confidence students was similar also. In examining differences in Feedback to the four groups, the patterns were roughly the same for all groups, except for negative Feedback. High confidence males received more negative Feedback than did other groups, and a larger proportion of all Feedback received by high confidence males was negative than for the other three groups. (See Figure 9)

Teacher Attribution of Student Success/Failure

Teachers made very few attributions to target students during the course of the observation (Table 1). Out of approximately 2500 interactions which occurred during 105 hours of observation, there were 21 attribution comments recorded from teachers. Fourteen of the attributions were for success and 7 were for failure. Over half (8 out of 14) of the success attributions were to ability, and over half (4 out of 7) of the failure attributions were to some characteristic of the task. For success, more attributions were made to boys than to girls, and more attributions were made to high confidence than to low confidence students. Most attributions for success were to high confidence males. The attributions for failure appear to be scattered somewhat randomly across sample group and type of attribution.

Summary

In summary the following ideas emerge:

(1a) Males were involved in more interactions than were females.

(1b) High confidence males were involved in the highest percentage of interactions of the four sample groups, while high confidence females were involved in the lowest percentage of interactions.

(2a) Females had more No-interaction days than did males.

(2b) High confidence females had more No-interaction days than any other

Figure 9

Feedback

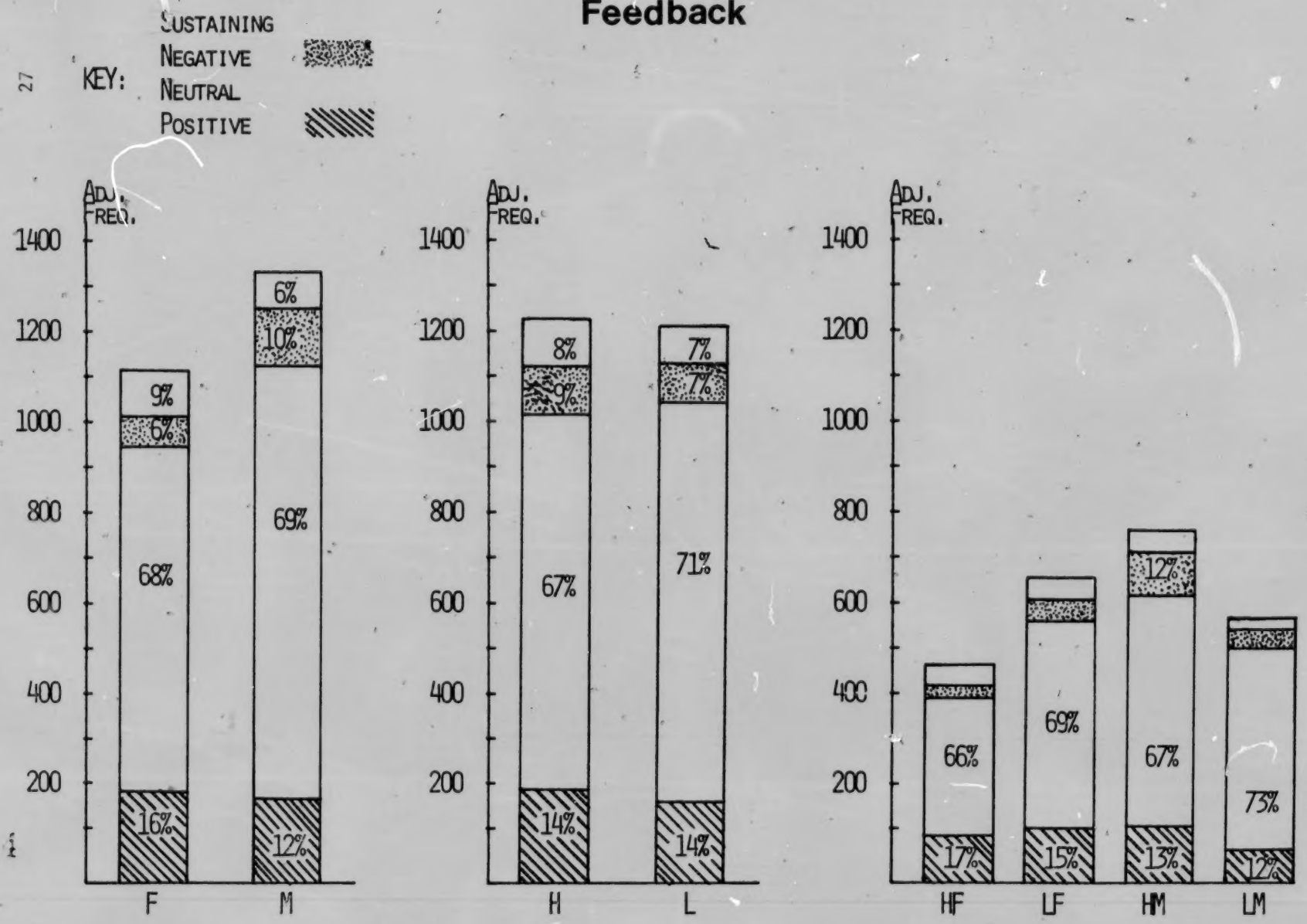


Table 1

Teacher Attributions of Student Success/Failure

Sex	Female		Male		
Confidence Level	High	Low	High	Low	Total
Success					
Ability	0	0	5	2	8
Effort	1	0	1	1	3
Task	0	1	1	0	2
Environment	1	0	0	0	1
Total Success	2	2	7	3	14
Failure					
Ability	0	0	0	0	0
Effort	1	1	0	0	2
Task	0	2	0	2	4
Environment	0	0	0	1	1
Total Failure	1	3	0	3	7

group, while high confidence males had fewer No-interaction days than any other group.

(3) There was no significant sex difference in the number of Mathematics interactions.

(4) Males had more Higher order (Process) interactions than females. One third of the Process interactions were with high confidence males, one third with low confidence females, and one third with high confidence females and low confidence males combined.

(5) High confidence females initiated 30% of their interactions while low confidence females, high confidence males, and low confidence males each initiated about 40% of their interactions.

(6) Teachers only rarely made attributions of the causes of student success or failure.

Conclusions

Taking into consideration only sex of student or only confidence level of student did not yield nearly as much information as when students were categorized by both sex and confidence. Confirmation for the use of confidence as a characteristic of students which influences patterns of teacher-student interaction has been obtained.

High confidence females and high confidence males had very different patterns of interactions with their teachers. However, from this study alone it is not possible to determine the causes of these differences.

Generally males had more interactions with teachers than did females. Males had fewer days when they had no interactions with the teacher than did females. Despite this consistent pattern in differential number of interactions to females and males, females and males in this study had approximately the same number of Mathematics interactions. Where males

and females differed was in their Non-mathematics interactions, with males involved in nearly twice as many Non-mathematics interactions as females.

Discussion

One stated objective of this study was to identify differences in the patterns of interactions between mathematics teachers and girls and boys with different confidence levels. Beyond this, it is hoped that over the three years hypotheses can be generated from these data concerning some of the causes of sex-related differences in student confidence in learning mathematics. At the present time the data have not been analyzed fully, but a variety of questions are raised from the analyses completed.

In mathematics classes, what are the long term effects of being involved with more or less interactions with the teacher than are other students? Might such differences in Total interactions have an effect on the development of students' confidence in their ability to learn and perform in mathematics? Might such differences have an effect on students' decisions to continue or not continue the study of mathematics?

Are Non-mathematics interactions important as an influence on student confidence? It is interesting that females and males differed only slightly in the number of Mathematics interactions they were involved in. However, for Non-mathematics interactions which are a combination of several types of interactions, males had nearly twice as many as females. Is this result a consistent one?

What are the long term effects of having no interactions with the teacher one out of four days, as is true for the high confidence females in the reported data? In light of the fact that most instruction in mathematics comes from the teacher in mathematics class, it may be important that some students often were present but had no interaction at all with

the teacher during entire class periods. This could conceivably influence both student confidence and achievement in mathematics.

What is happening to cause the pattern of interactions found for high confidence females? Is it that high confidence females have less need to interact with the teacher than do high confidence males? Or do high confidence females sense that it is not appropriate for females to interact with their teacher concerning mathematics? Do teachers in some way discourage interactions with high confidence females? Or is it that females respond differently to the Fennema-Sherman Confidence scale than do males, such that a high score in confidence means something different for females and males?

Teachers made few attribuions of student success or failure. When teachers so rarely attribute student success/failure to a particular cause it seems unlikely that teachers have much influence on students' patterns of attributions, or that teachers promote learned helplessness in some students but not others.

CONFIDENCE INTERVIEW

This paper describes and reports the results of the second approach to the investigation of the relationship between confidence and the learning of mathematics. Although confidence in learning mathematics has been shown to relate to both achievement and election of mathematics courses, little is presently known about the factors that influence the development of confidence with respect to students' perceptions of themselves as learners of mathematics.

The dictionary (Webster's New Collegiate Dictionary, p. 237) defines confidence as '... a feeling of consciousness of one's powers ...' '... in one's ability to succeed.' Thus, questions come to mind. Do male and female students have different expectations with respect to girls' and boys' successes and failures in mathematics? Do they respond differently, do they feel differently about girls' success, boys' success? To what factors do they attribute the successes of girls and of boys? Are these attributions different? Are they the same? How do students' perceive their teachers' beliefs about them? Do they feel teachers have differing success or failure expectations with respect to their girl and boy students? Do students of different confidence levels have differing patterns of responses to these kinds of questions? How about different kinds of math problems? Do students feel girls or boys are more likely to be successful in solving math problems of lower or higher levels of difficulty?

These questions may be summarized within three groups ...

1) What are expectations of success and failure in mathematics by girls and boys?* What are girls' and boys' perceptions

* Girls and boys are referred to below as the 'target group' to differentiate them from the subject females and males who are the 'responders.'

of expectations of both sexes successes/failures by their peers and teachers?

2) What are girls' and boys' feelings about successes/failures in mathematics and how do they perceive that peers and teachers feel toward girls' and boys' success/failure?

3) How do boys and girls attribute causation of successes/failures and do they perceive attributions by peers and teachers to be different from their own?

It is these three categories ... expectations, feelings, and causal attributions ... each with respect to subject/peer/teacher, success/failure, process/product, and spatial/non-spatial that form the main dimensions of this study. Figure 1 is a graphic representation of the areas investigated.

PROCEDURE

The Sample

High and low confidence females and males were selected as previously described. The sample included in this portion of the study consists of seventeen high confidence females, twenty-two low confidence females, twenty high confidence males and fifteen low confidence males.

The Instrument

Since the major objective of this part of the study was to generate rather than test hypotheses, a structured interview using a projective technique was selected as an instrument most likely to allow the greatest range of student response to the questions being raised. This decision was

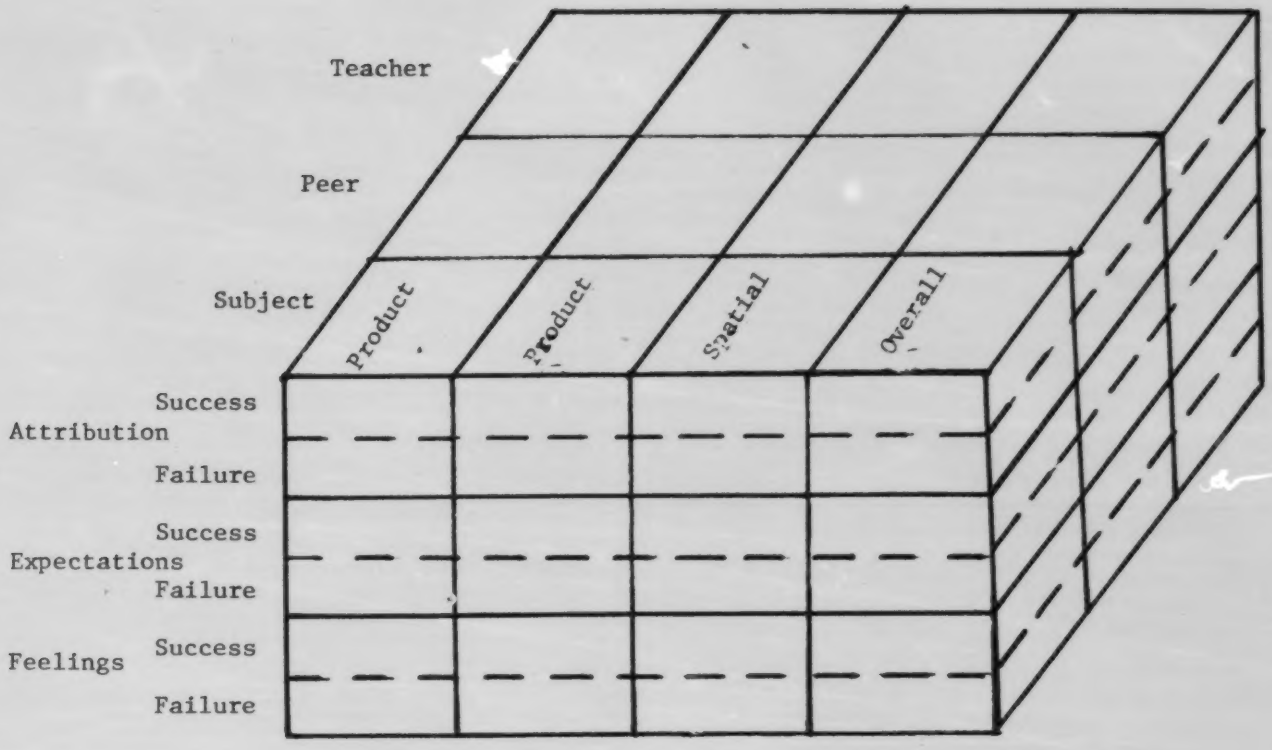


Figure 10
Model for Interview Variables

made after several pilot interviews, in which students were asked direct questions about their perceptions of themselves as well as their peers and teachers in mathematics classes, failed to yield satisfactory results. Pilot testing using the instrument described below proved effective in causing students to speak more freely about their perceptions of mathematics classes, and was thus adopted instead.

The instrument used in this study consists of eight photographs accompanied by specific questions. These photographs are pictures of female and male students (of the same age as the subjects), and in some cases, teachers as well, in groupings representative of mathematics classrooms. During the course of the interview students are shown photographs and asked to respond to a sequence of questions on the one or more cards accompanying each picture (Figure 11). Questions are designed to provide information to certain categories by eliciting student responses to the situations in the pictures. For example, in the following question asked about the picture shown in Figure 2 ... "Who among these four does the teacher think will know how to do the problem?" the subject makes a sex choice about a teacher's success expectation with respect to a spatial problem. To the question "What reasons would the student give for being able to do the problem" the subject is making a 'self' attribution with respect to success on a spatial problem ... and so on.

During the development stage of the instrument, pictures and questions were modified and refined in response to pilot interviews with sixth grade

HERE'S PICTURE 7: BOTH

7A

	THE TEACHER HAS ASKED A PROBLEM ABOUT THE AREA OF THE WALLS OF A ROOM (Show problem 7A & B and read it.)
TEACHER	Who among these four does the <u>teacher think</u> will know how to do the problem?
EXPECTATIONS	Why does the teacher think that the student can do it?
&	How would the student feel?
ATTRIBUTIONS	What would the teacher say and think about her/him?
&	What reasons would the teacher give for her/his being
STUDENT	able to do it? (Attribution)
ATTRIBUTION	What reasons would the student give for being able to do it? (Attribution)

Figure 11

Photograph and Accompanying Questions

(Use same photo for next card)

students. An attempt was made to control any elements in the photographs that might confound students' responses to the pictures. (For example, we worried about some student in a group of four looking 'smarter' or indisputably 'more alert' than the others, thus, eliciting a disappropriate number of student selection as most likely to get the right answer.)

Interviewers from the project staff were carefully trained to use the interview instrument. Time was spent talking about the purpose of the interview and the proper approach to take in dealing with students so as to encourage students to give their own feelings rather than to reflect the opinions and ideas of the interviewer. The questions were printed on 5 x 7 cards and numbered so they could be ordered easily. The questions to be asked with each picture were read and discussed. Once the interviewers were familiar with the technique, they practiced it with 1-2 sixth grade students in a pilot school.

Each interview lasted 30-45 minutes and was held with one interviewer and one student in a private place in the school the student attended. The interviewer did not know the Confidence level of the student interviewed. Parent permission was received before students were interviewed. All interviews were carried out between April 23 and May 30, 1979.

Interview tapes were transcribed and coded. Expectations were coded male or female depending on the sex of student to whom the success or failure was ascribed. Responses to questions about feelings were coded as negative, neutral, or positive; responses to attribution questions (i.e. Why do you think she/he got the problem right?) were coded as ability ("she was smart," "he always gets these right"), effort ("he works hard," "she concentrates"), task ('it was an easy problem'), or environment ("got help from teacher/ parent," "was lucky").

There were some post-hoc categorizing decisions. For example, in cases where it was found that a considerable number of interviewees offered attribution responses to questions where no such responses had been anticipated, the decision was made to expand the coding to include such attributions. Thus, responses of students who had not offered attributions in these categories were coded as missing data, generating a larger amount of 'so called' missing data than is actually missing.

Coding agreement was established by comparing independent codings of a selection of transcripts by the two researchers who developed the scheme, and then on another sample, between the four researchers who were to do the final coding. Each researcher coded a random fourth of the data with some interviews coded by all, on which coding reliability was quantified.

Analyses and Results

The major technique used in this study was log-linear analysis, designed to generate for multi-dimensional frequency data the same levels of information previously available for the analysis of continuous data. Log-linear analysis (Everitt, 1977; Haberman, 1977) tests for the significant contribution of main effects in the study, as well as for interaction effects at all levels. This information helps guide the researcher to an understanding of the relative importance of various effects and the interrelationships between them, within the problem under consideration.

Frequencies adjusted for differences in cell size were used in the analyses. Reported here are the results of the analyses which indicated that the frequencies observed probably did not occur by chance. Although in all cases the analyses were performed with the observed frequencies

reported in the tables below, the data in the bargraphs have been converted to percents in order to clarify comparisons between groups on significant effects.

In the discussion below, in order to distinguish between them, the sex of the responder group is described as female and male while the sex of the target group is referred to girl and boy.

Expectations

This part of the anlaysis deals with responses to three groups of questions. In questions such as... "Who do you think would probably have the right answer?" and "Which student would the teacher pick as liely to know how to do that?" subjects selected the girl or boy in the picture whom they perceived as most likely to be successful. These responses generated the expectation of success frequencies seen in Table 2.

In questions such as ... "How might the others expect this girl to do on their math lesson?" or "How might the others expect this boy to do on their math lesson?" subjects were asked to choose between success and failure expectations for students of specific sex. These data are presented in Table 3.

Finally, "Who among these four does the teacher think will know how to do the problem?" a questions about a spatial problem, was one where subjects were asked to choose both sex and success versus failure for the students in the picture. The responses to questions such as those generated the data in Table 4.

Figures 12, 13, and 14 summarize the significant effects from the data in Table 1. Figure 12 shows that subjects overall express more success expectations of girls than of boys. Figure 13 shows that this behavior is different for females than males. Females make such choices at a rate of almost 2:1 whereas males do so at a much lower rate. Figure 14 indicates that the main differences between the high and low confidence females, with low confidence females expecting more success for girls.

From these first analyses then, we can say that both females and males, in our sample, expect girls to be more successful than boys; females feel this way more so than do males, and low confidence females have the highest expectations for girls.

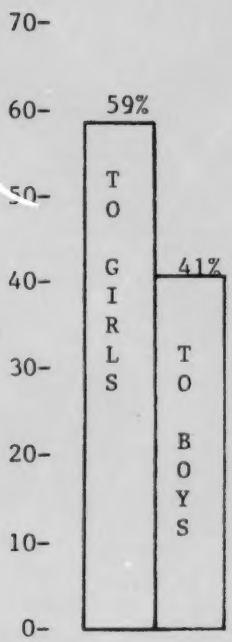
Table 2

Success Expectations by High and Low Confidence Females and Males

Target Group	Responder Group: Female Hi Conf.	Female Lo Conf.	Male Hi Conf.	Male Lo Conf.
Girl	71	97	87	66
Boy	46	52	70	53

Figure 12

Success Expectations to Girls versus Boys, over all groups (Table 2)



$\chi^2 = 18.56$

$p < .001$

Figure 13

Success Expectations to Girls versus Boys by Femals and Males (Table 2)

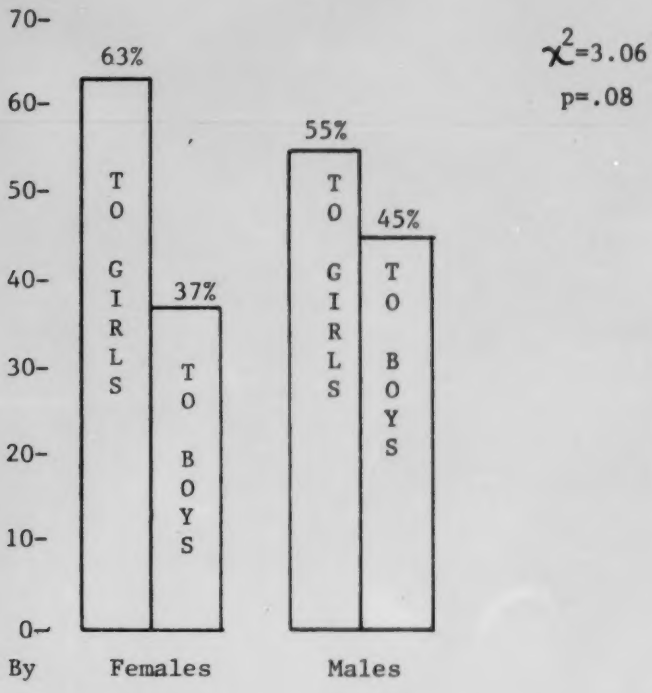
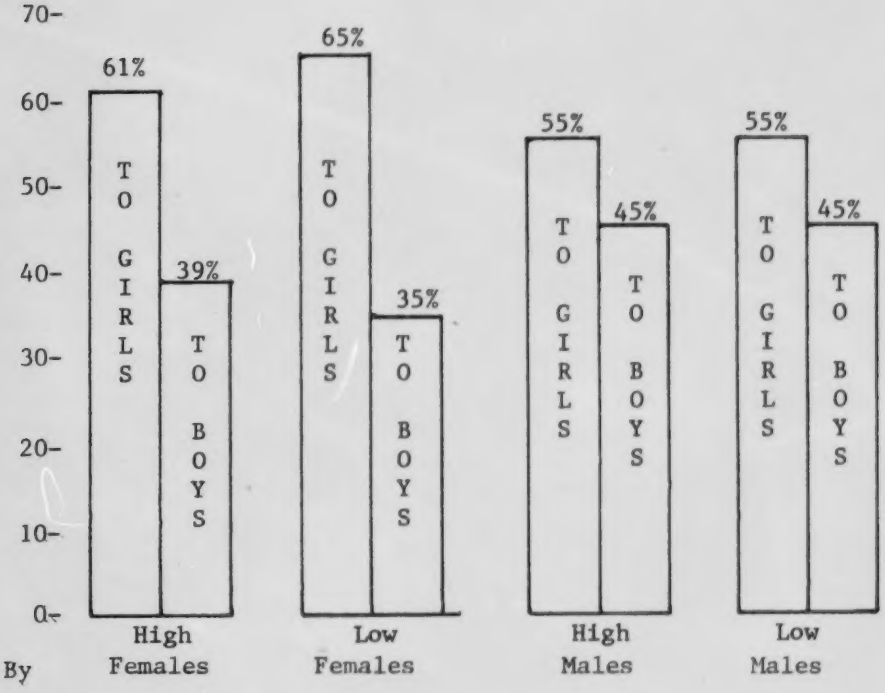


Figure 14

Success Expectations to Girls versus Boys by High and Low Confidence Females and Males (Table 2)



In two questions, students were asked to express success or failure expectations with respect first to a girl, then a boy. The frequencies in Table 3 are responses to the questions ... "How might the others expect this girl (later,"this boy") to do on their math lesson?" Table 2 shows the frequencies of success versus failure expectation responses to these questions. For example, sixteen low confidence males predicted success on the first question, while four high confidence females predicted failure. On the other hand, fourteen high confidence males predicted success when asked the same question about a boy. Among both high and low confidence females as well as high confidence males, four students in each of these three groups predicted failure when asked this question about a boy.

The analysis of these data indicates that there are no statistically significant differences with respect to the research question ... "Are there any differences in students' expectations with respect to girls or boys?" All groups demonstrate essentially the same expectations about both sexes. However, these results, seen graphically in Figure 15, suggest some interesting trends. Note that high confidence females express the lowest rate of success expectations about girls ... whereas the low confidence males expect the highest ... in fact 100% ... success for girls.

One picture in the instrument was directed toward teachers' expectations with respect to success or failure, on a spatial problem.* Table 4 shows the frequencies of responses to these questions.

From Table 4 we see that overall, all groups perceived the teacher as expecting a greater number of success (48) than failure (25) responses. Further, more success than failure expectations were attributed to teacher

*How many square meters are there on the walls of a room whose floor is 5 meters by 4 meters and whose height is 3.5 meters?

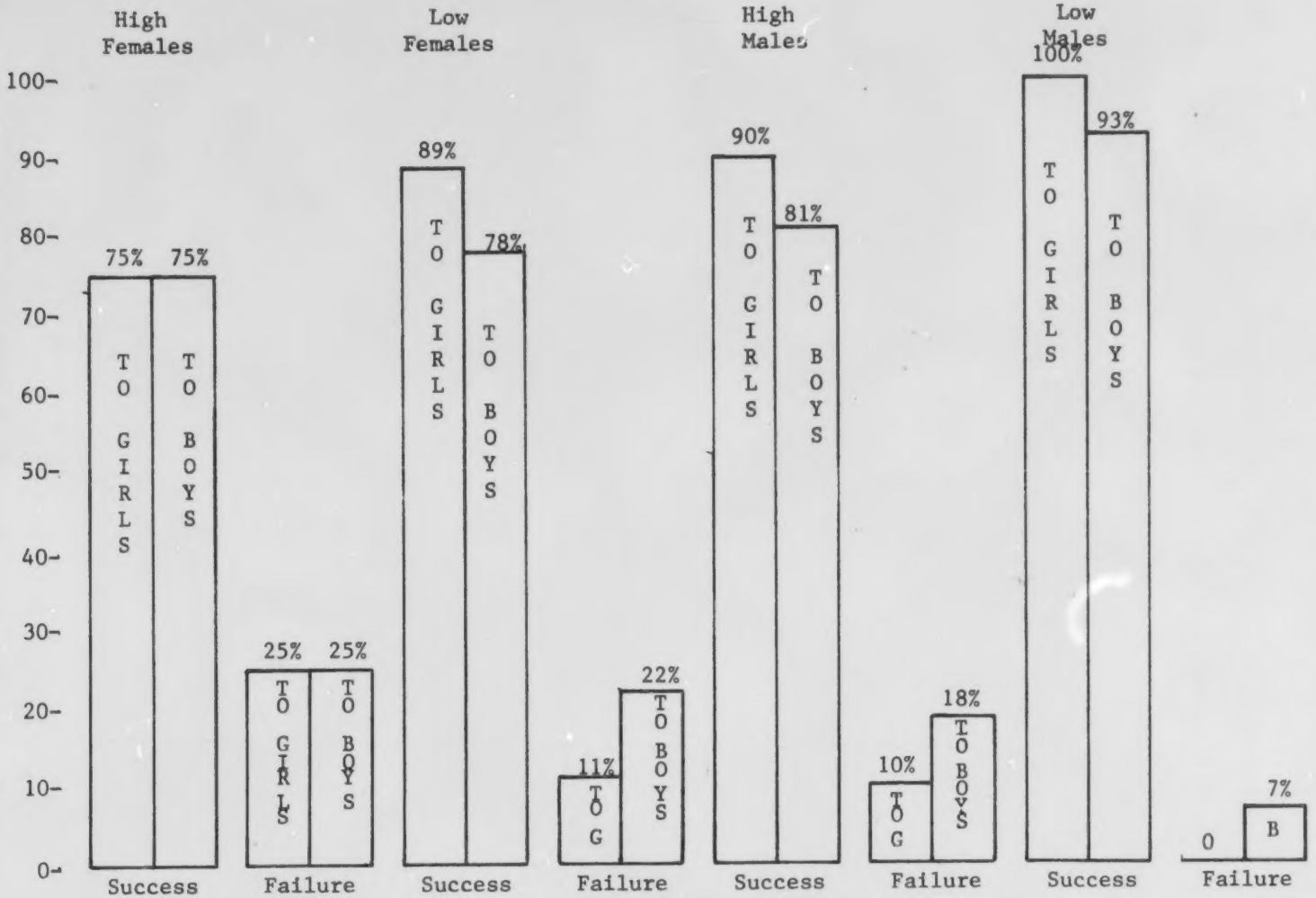
Table 3

Expectations with respect to
Girls' and Boys' Success or Failure

		Responder Group			
		Female		Male	
Expectation	Target Group	Hi Conf.	Lo Conf	Hi Conf.	Lo Conf.
Success	Girl	12	17	19	16
	Boy	12	14	17	14
Failure	Girl	4	2	2	0
	Boy	4	4	4	1

Figure 15

Comparison Between Success versus Failure Expectation Expressed Toward Girls and Boys by High and Low Confidence Females and Males (Table 3)



with respect to both girls and boys. Figure 16 shows a significant success/failure by target sex interaction. There is a greater success/failure ratio expected for girls, by all students, than that expected for boys. Figure 17 shows a slightly significant four-way interaction between groups on success/failure expectations toward girls versus boys. The main item of interest here seems to be the low confidence boys who expect boys to be successful at a greater rate than girls.

In summary then, students perceive teachers as expecting more successes than failures and they believe this is the case for both girls and boys. However, the rate of success expected for girls is greater than that expected for boys by all groups except low males. Since the teacher expectations are, of course, subjects' perceptions of teachers' expectations, we see again that most students in our sample expect girls to be more successful than boys in mathematics. The exception here is the low confidence boys who indicate a much lower rate of teacher success expectation for girls than they did for the questions in Table 4.

Feelings

The second major category investigated in this study is students' feelings about success and failure with respect to learning mathematics. Here, questions were asked which were classified according to respondent sex, confidence; target sex; subject-peer-teacher, problem type, and success/failure. For example, "How did the one who did not do well feel?" and "How did the one who did well feel?" would be classified as subject-feelings about a girl's or boy's failure or success feelings, respectively. The question "How will this one (point to one who got it right) feel about the one who got it wrong?" would be coded as a peer response to failure. "When

Table 4

Students' Perception of Teacher Expectation with Respect to Success or Failure of Girls or Boys

		Responder Group			
		Female		Male	
Expectation	Target Group	Hi Conf.	Lo Conf.	Hi Conf.	Lo Conf.
Success	Girls	3	5	9	2
	Boys	9	8	5	7
Failure	Girls	1	1	1	2
	Boys	4	7	5	4

Figure 16

Students' Perceptions of Teacher's Expectation with Respect to Success or Failure of Girls or Boys (Table 4)

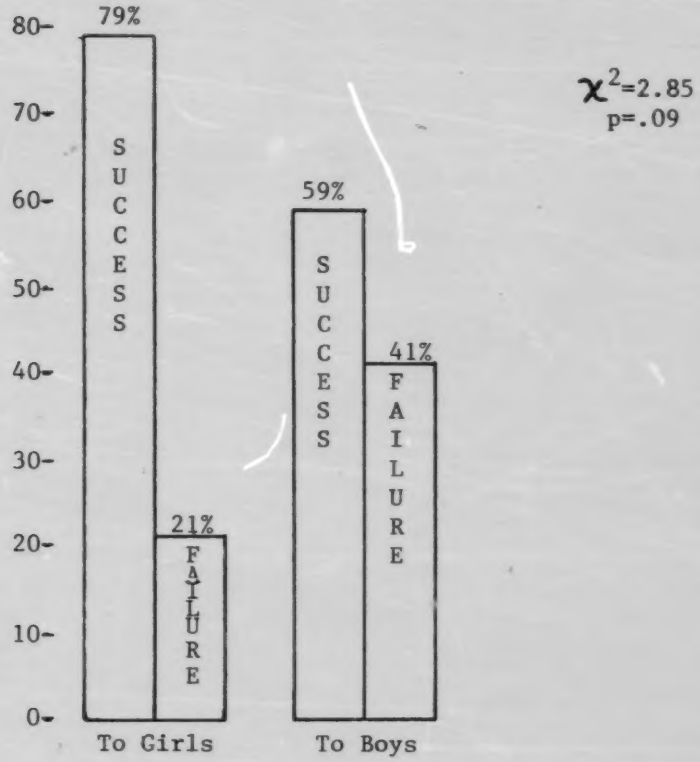
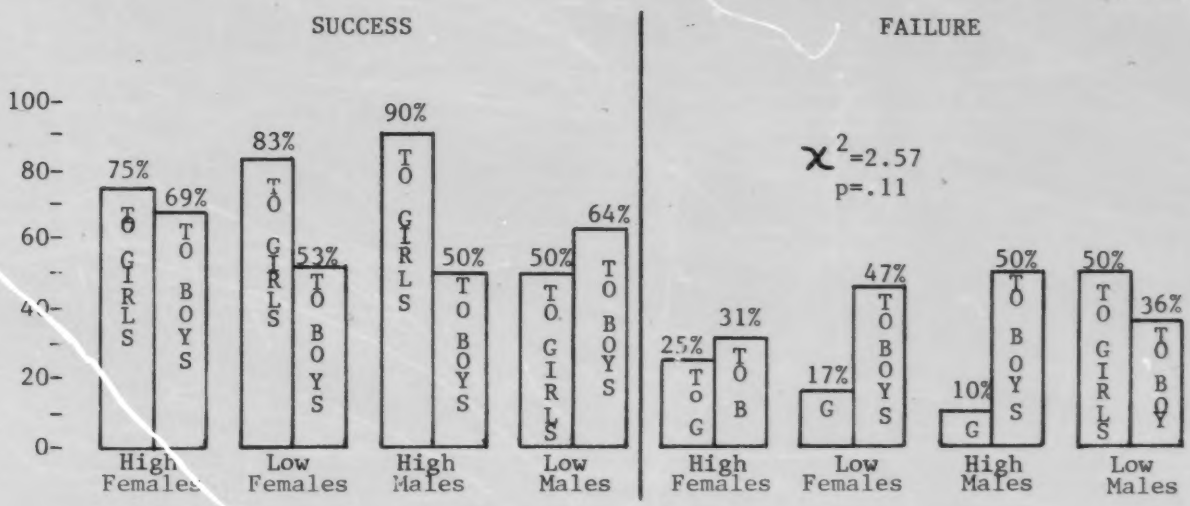


Figure 17

High and Low Confidence Females and Males Perceptions of Teacher's Expectation with Respect to Success or Failure of Girls or Boys (Table 4)



the student has answered correctly, what would the teacher think about the student" ... is a question that would be coded as teacher's feelings about a girl's or boy's success with respect to a process, product, or spatial problem (depending on which problem type was designated for the picture associated with the question.) The frequencies for each category are in Table 5.

Figure 18 shows an interesting effect. As one would expect, all students feel worse about failure than about success. This distribution of negative to positive feelings is the same toward girls and boys with respect to success responses. However, the failure responses yield significant differences. Comparison between levels of feelings (negative, neutral, and positive) with respect to girls' and boys' failure indicates a more negative reaction to boys' failures than to girls'. Note again that the equivalent ratios with respect to feelings about success (positive:negative) are essentially the same toward girls and boys.

Figure 19 shows a significant interaction between sex, feelings, and the success/failure dimension. Here the interaction between sex and feelings is significant for both success and failure, although the effect is more pronounced with respect to failure feelings. In both cases, females seem more negative than males. They are less positive with respect to success and at the same time, more 'down' on failure than males.

In Figure 20 a significant interaction is seen between success feelings and confidence group. High confidence students express less positive feelings with respect to success than do low confidence students. This interaction is not significant with respect to failure feelings, where the patterns for both groups are virtually identical.

Table 5

Success and Failure Feelings with Respect to Target Group and Subject-Peer-Teacher

Subject Feelings

Feelings	Confidence	Female High			Female Low			Male High			Male Low		
	Target Group	-*	0*	+*	-	0	+	-	0	+	-	0	+
Success	Girls	3	5	34	2	11	50	2	8	30	0	7	33
	Boys	3	5	19	1	8	22	2	8	29	0	3	28
Failure	Girls	7	11	0	13	3	0	9	6	0	8	2	2
	Boys	19	4	0	21	12	1	14	6	0	14	4	6

Peer Feelings

Feelings	Confidence	Female High			Female Low			Male High			Male Low		
	Target Group	-	0	+	-	0	+	-	0	+	-	0	+
Success	Girls	11	2	7	4	7	12	6	6	11	5	2	5
	Boys	3	1	8	3	5	5	3	4	4	2	1	8
Failure	Girls	10	9	0	11	11	1	8	13	1	4	11	1
	Boys	5	4	0	3	8	0	6	5	3	6	2	1

Teacher Feelings

Feelings	Confidence	Female High			Female Low			Male High			Male Low		
	Target Group	-	0	+	-	0	+	-	0	+	-	0	+
Success	Girls	0	3	30	0	11	38	0	12	29	0	6	26
	Boys	1	4	12	0	5	17	0	6	25	0	3	15
Failure	Girls	3	2	0	2	3	0	0	6	0	3	1	0
	Boys	9	4	2	9	12	0	7	3	0	9	3	0

* - = negative, 0 = neutral, + = positive

Figure 18
Feelings About Girls' and Boys' Successes and Failures (Table 5)

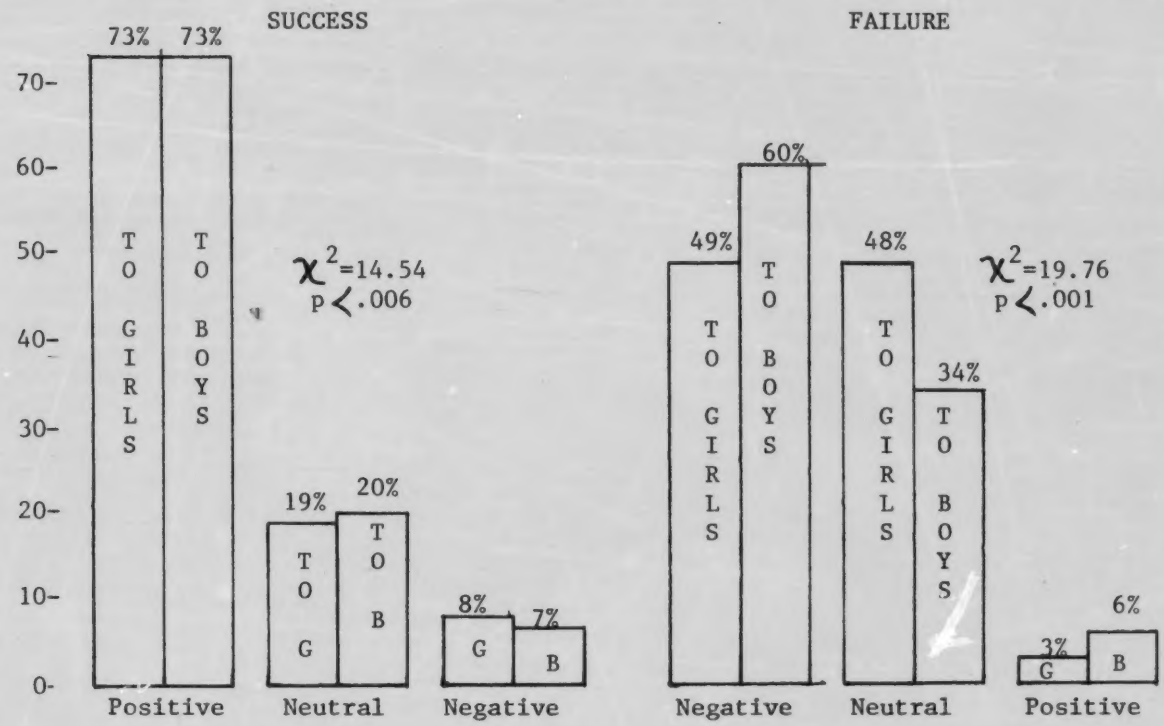


Figure 19

Feelings Toward Success and Failure by Females and Males (Table 5)
(For both success and failure the neutral responses are approximately the same)

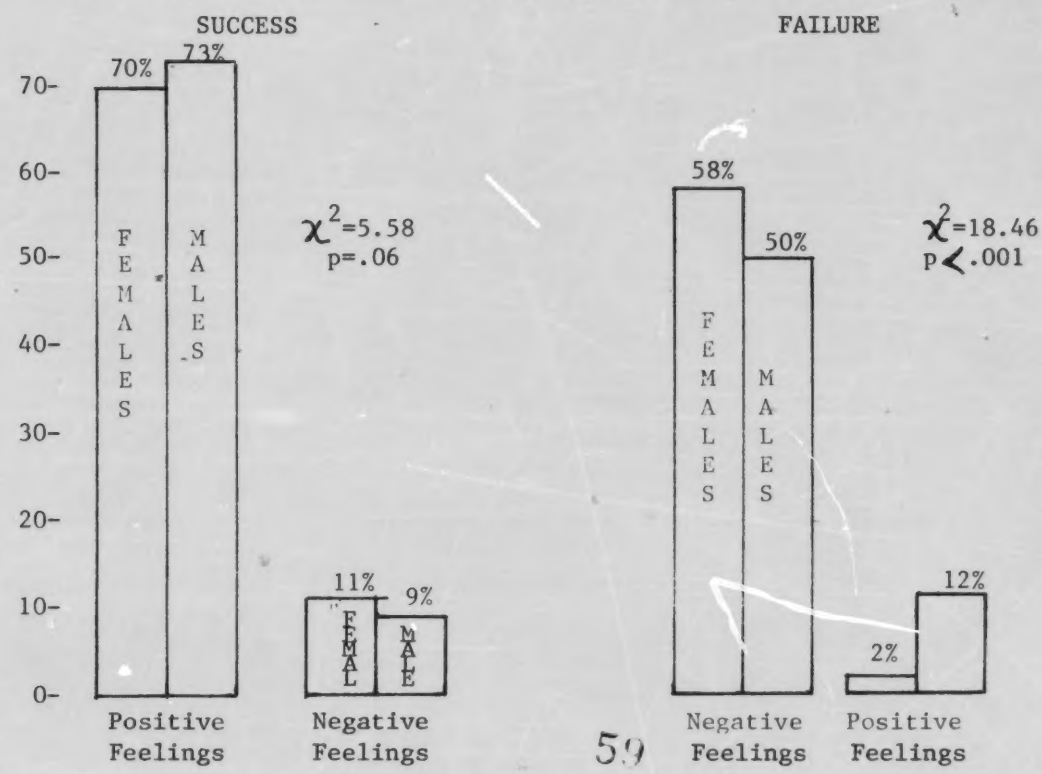
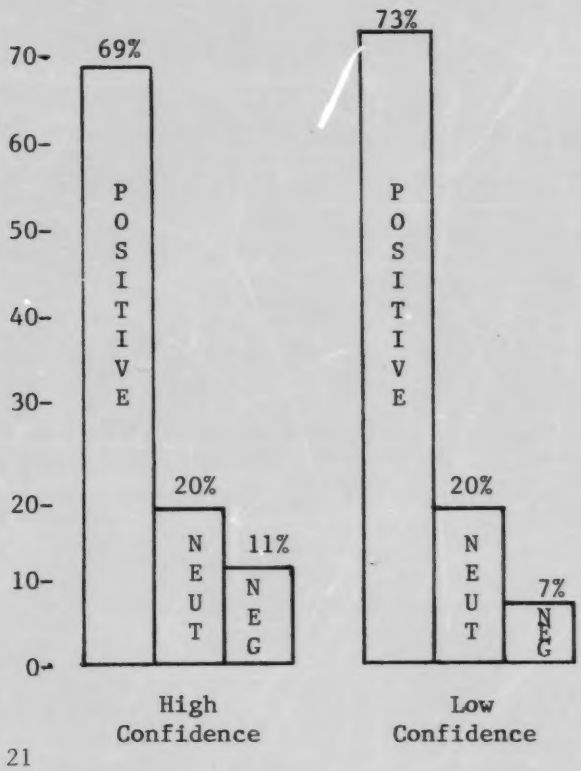


Figure 20

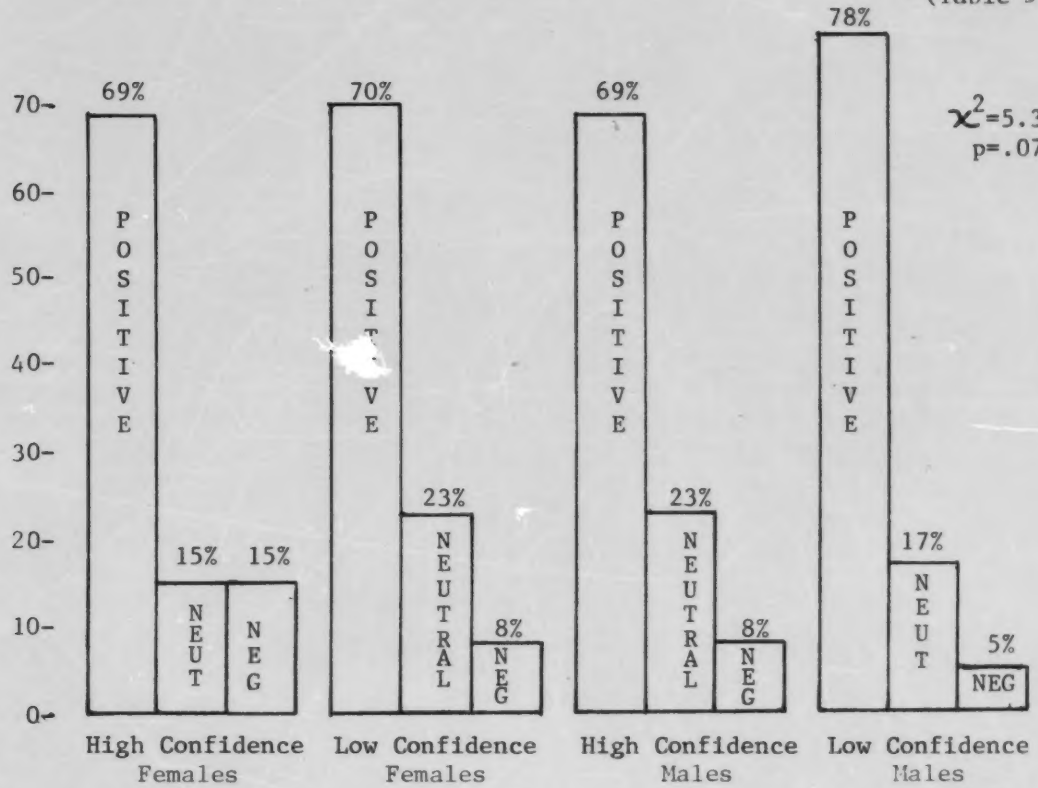
Success Feelings Expressed by High versus Low Confidence Students (Table 5)



$\chi^2=6.56$
p=.04

Figure 21

Feelings About Success by High and Low Confidence Females and Males (Table 5)



$\chi^2=5.38$
p=.07

Figure 21 shows a three-way interaction between confidence, sex, and feelings which is significant with respect to success. Here again we see a consistent pattern...although females share the same positive feelings toward success as do high confidence males (note low confidence males are more positive), high confidence females tend to be more negative about success.

So far, a picture is emerging of females who tend to have less positive feelings about success and more negative feelings toward failure. Males, on the other hand, are less negative about failure; in some cases, even positive about failure (see Figure 19 .

Figure 22 shows a significant result when subject-peer-teacher is included as a categorical variable. Here the frequencies for feelings, subject-peer-teacher generate a significant interaction for both success and failure. Subjects perceive their own and teachers' responses to success as similar; they see different responses, much less positive, from peers. This effect is the same with respect to failure feelings; that is, a fairly clear identification of teachers' feelings with feelings of self ... and both of these are different from feelings attributed to peers.

A slightly different 'feelings' category is formed by response to the question, "How does the student feel about coming for help from the teacher when having trouble?". These responses are shown in Table 6. The analysis of these data indicate that although none of the main effects (confidence or sex or feelings alone) are significant, all the two way interactions are; in particular both confidence x feelings and sex x feelings are significant at the .01 level.

Figure 23 shows the interesting result that females are more negative about going to the teacher for help than males. We see this same result

Figure 22

Students' Perceptions of Subject-Peer-Teacher Feelings With Respect to Success and Failure (Table 5)

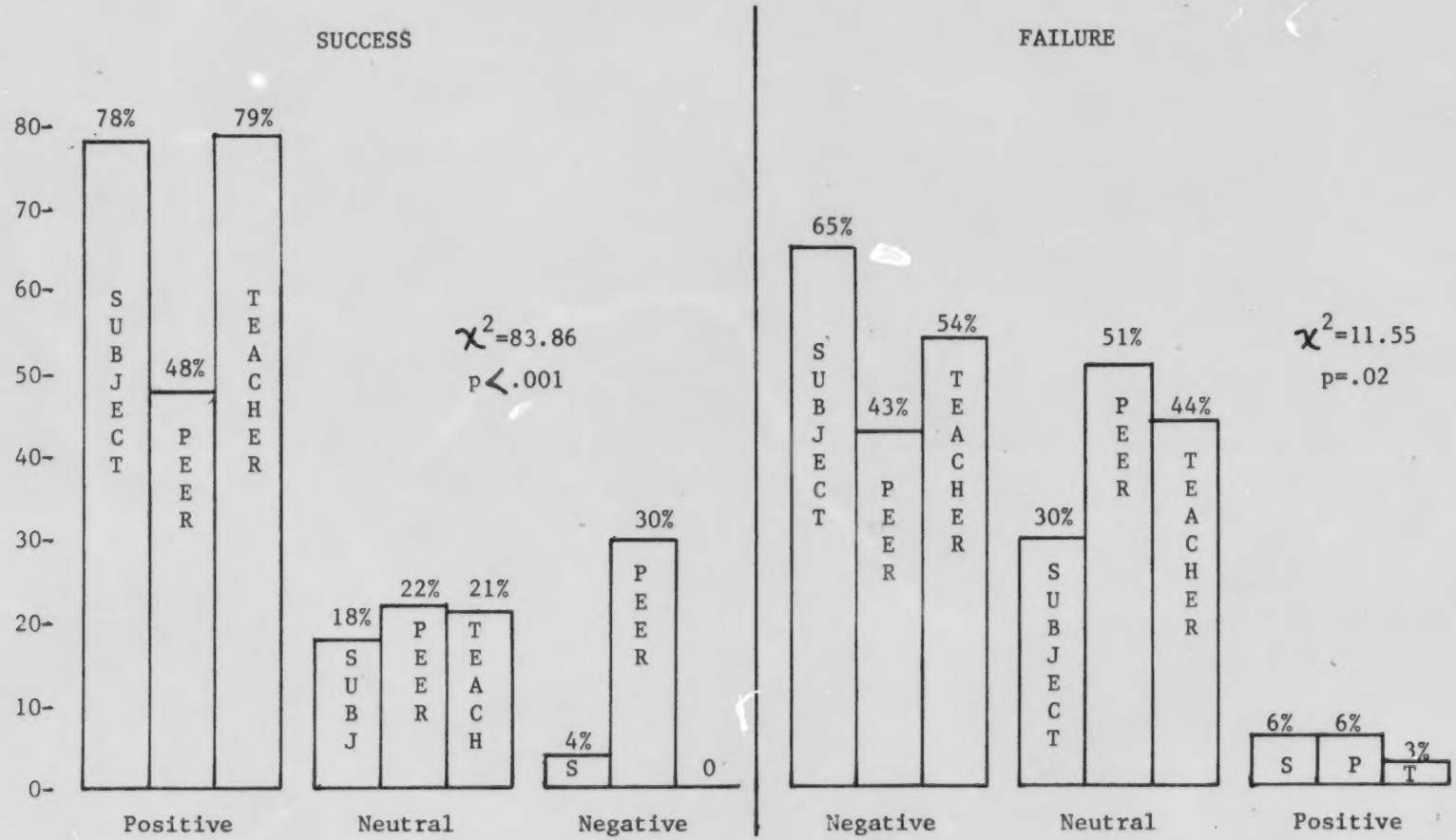


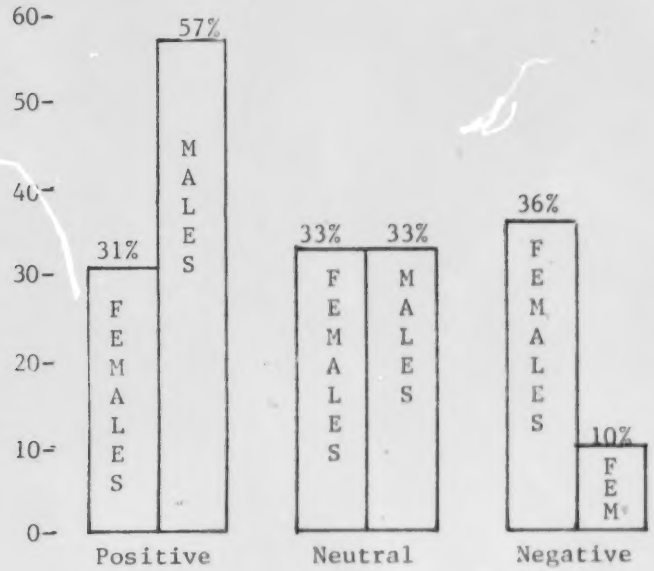
Table 6

Feelings of High and Low Confidence Females and Males about Coming to the Teacher for Help.

	Respondent Group			
	Female		Male	
Feelings	Hi Conf.	Lo Conf.	Hi Conf.	Lo Conf.
Negative	11	3	2	1
Neutral	3	10	5	5
Positive	3	9	10	7

Figure 23

Feelings About Going to Teacher for Help by Females and Males (Table 5)



$\chi^2 = 10.50$
$p = .005$

with respect to confidence groups. Figure 24 shows high confidence students to be more negative about asking the teacher for help than low confidence students. Even more striking is this result as it appears in Figure 16. High confidence females are least happy about asking the teacher for help. This is the same group which was found to be involved in the fewest number of teacher interactions in the classroom observation study. This result may be an important one in unraveling why fewer females than males go on in mathematics. Such an attitude would tend to isolate this group, the one least likely to avoid mathematics for any reasons that might result from low confidence, from help at a crucial time, a time when mathematical ideas become more complex and difficult to understand.

Attribution

The attribution analyses deal with data generated within the categories, sex, confidence, target sex, problem type, attribution pattern, and success/failure. These data are compiled from responses to questions such as the following. "Why do they think that the others did well." Response to this question would be coded as a success attribution by a peer to a girl or boy. "What reason would the teacher give for why the student got the right answer" responses would be coded as a teacher success attribution about a girl or a boy. Responses to "What reasons would the student give for being able to do it (a spatial problem)" would be coded as a success attribution by self with respect to a spatial problem, for a girl or boy (depending on the sex previously selected for success). Table 7 is the tabulation of this data within the categories enumerated above. Figure 26 shows again that all subjects make more success attributions to females than to males. Here we see that this pattern, of more success attributions to females, is consistent across all problem types.

Figure 24

Feelings About Going to Teacher for Help by
High and Low Confidence Students (Table 6)

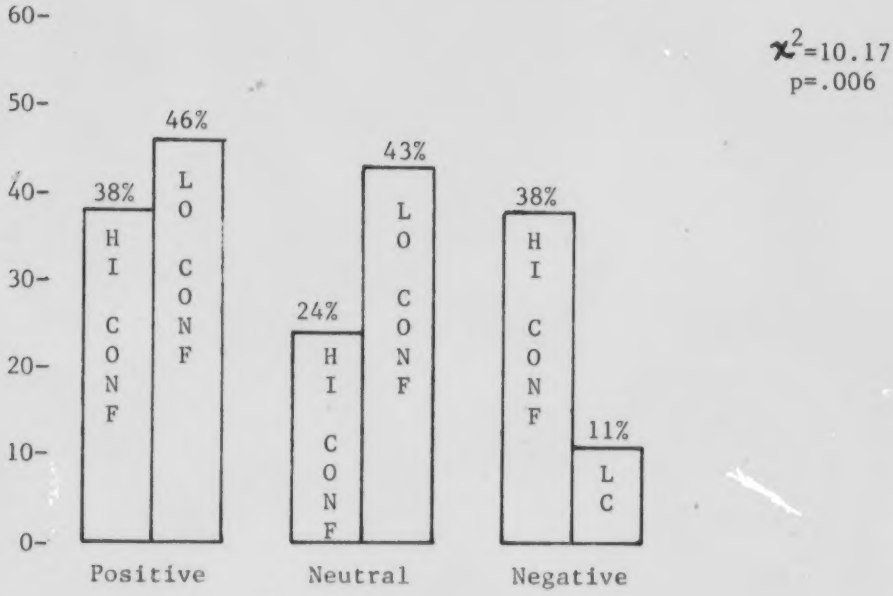


Figure 25

Feelings About Going to Teacher for Help by
High and Low Confidence Females and Males (Table 6)

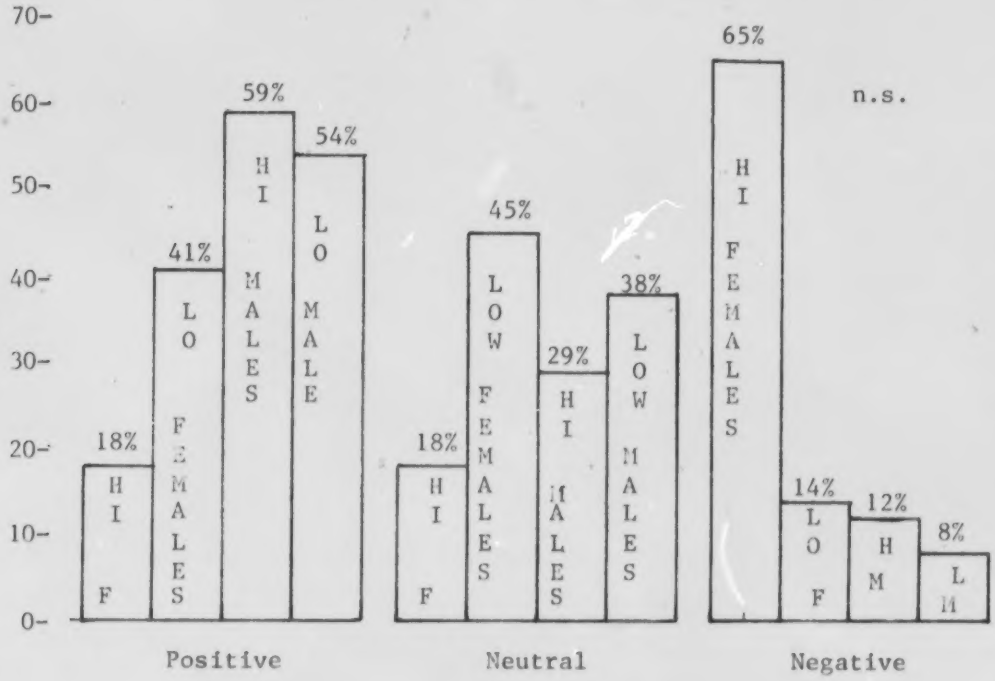


Figure 26

Comparison of Success Attributions Made to Girls and Boys by Problem Type (Table 7)

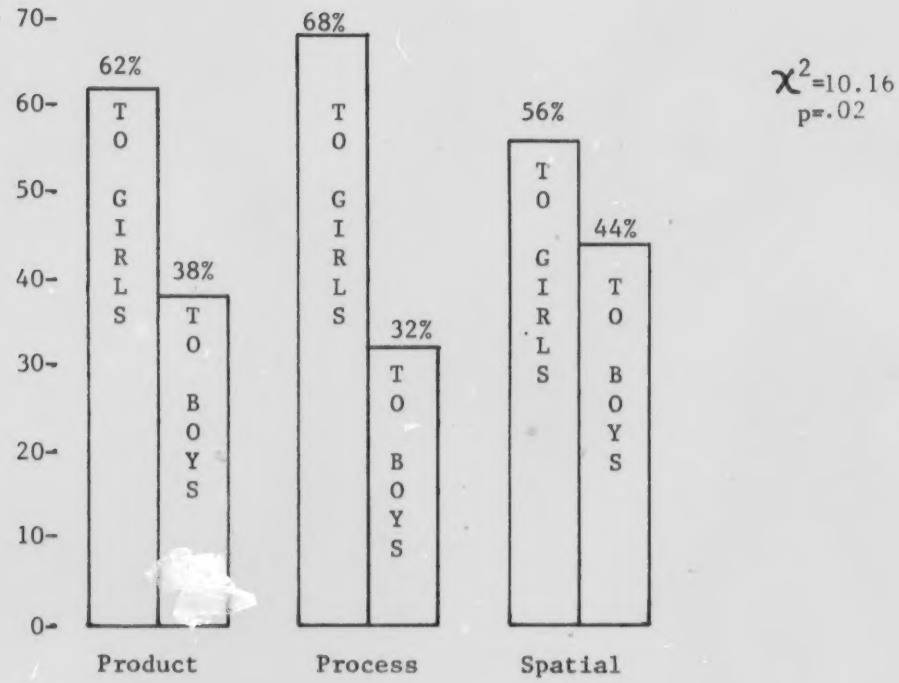


Figure 27 shows a significant attribution x problem level effect for both success and failure attributions. This effect seems to be due to a spatial, non-spatial contrast since it is no longer significant, for either success or failure, when the analysis is repeated excluding spatial problems. For all three problem types, more attributions are made to effort than to any other category. Further, the proportions between all four attribution categories seem roughly the same for product and process problems. The spatial attributions, however, are different. This difference is more apparent when the data are grouped in Figure 28. Here, when the ratios of unstable (effort and environment) versus stable (ability and task) dimensions of the attribution model are compared, the contrast between spatial and non-s½atial attributions is quite clear. This difference is even more striking for the failure attributions where the proportion of unstable to stable attributions is even greater than that for success attributions.

When the success attributions are analyzed alone, further effects become apparent. In Fiture 29, we see a significant interaction between confidence and patterns of attributions. Although both high and low confidence students make approximately the same numbers of effort and environment attributions (the unstable dimension) their distributions of ability and task attributions (the stable dimensioN) is different. We see high confidence students attributing more success to ability and low confidence students attributing more to task. Also, low confidence students show a greater proportion of effort to ability attributions then do high confidence students. This effect is not present with respect to failure attributions, and is present at a lower level of significance when the data are analyzed for product and process problem types alone.

When we look at the patterns of success attributions by sex in Figure 30, we see that although girls and boys show approximately the same ratio of effort to ability attributions, their attribution patterns differ on the

Figure 27

Distribution of Success and Failure Attributions by Problem Type (Table 7)

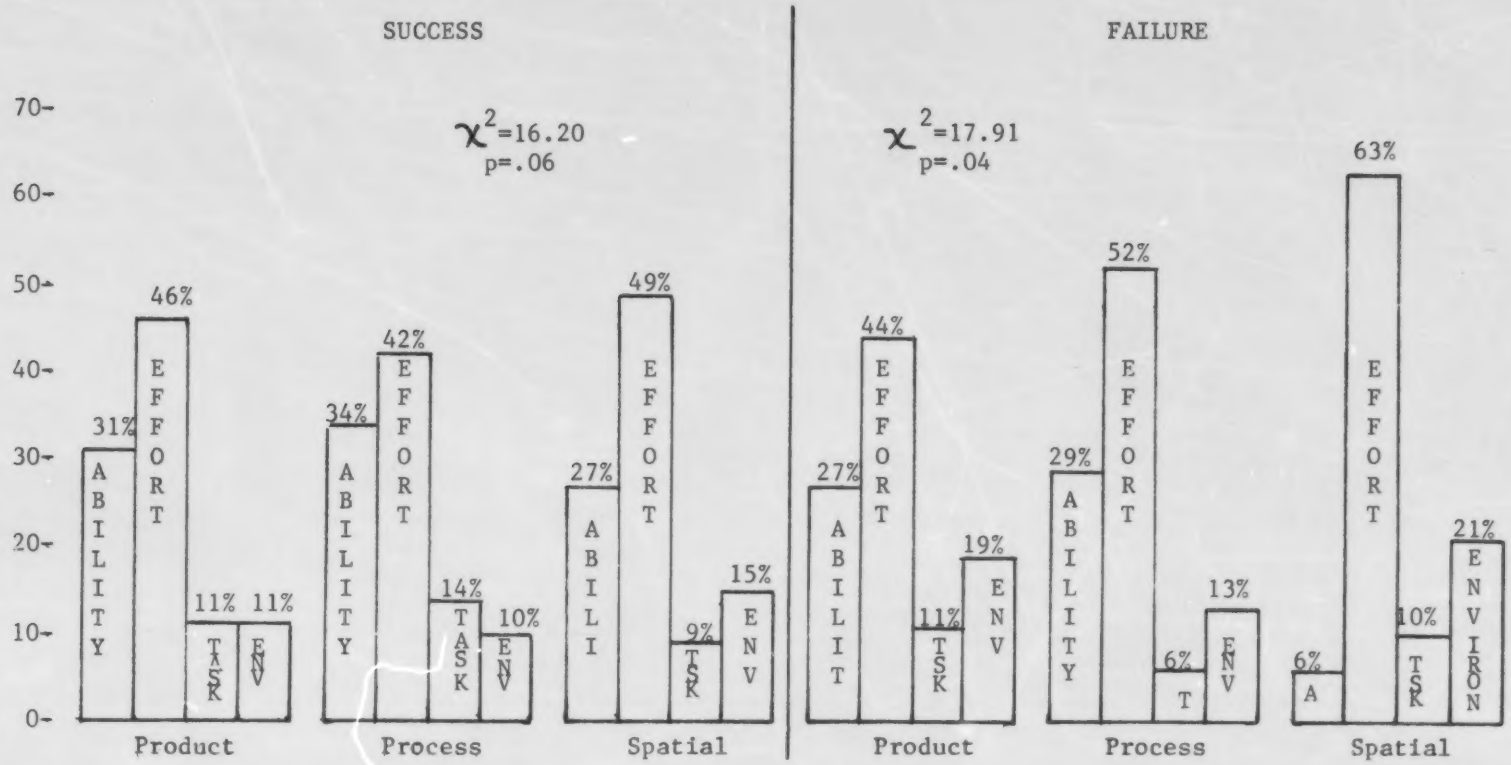


Table 7

Success and Failure Attributions for Problem Types to Target Students by High and Low Confidence Females and Males

			Female						Male					
		High			Low			High			Low		
Attribution	Target Group	P^1*	P^2*	S*	P^1	P^2	S	P^1	P^2	S	P^1	P^2	S
SUCCESS													
Ability	Girls	11	16	16	22	21	15	23	18	21	16	10	7
	Boys	9	5	8	10	7	8	21	10	12	3	8	7
Effort	Girls	27	18	21	27	28	26	27	21	18	26	13	28
	Boys	15	3	18	13	9	19	21	12	25	18	12	15
Task	Girls	4	7	3	11	10	8	5	3	2	6	3	0
	Boys	1	2	3	10	7	5	4	5	3	2	2	6
Environment	Girls	7	4	6	10	3	11	8	7	7	5	7	4
	Boys	6	2	5	4	1	8	5	3	5	0	1	5
FAILURE													
Ability	Girls	4	4	0	2	3	0	2	4	0	1	0	0
	Boys	9	11	0	8	11	1	4	6	1	3	1	1
Effort	Girls	5	5	2	6	5	3	3	6	1	2	5	2
	Boys	8	17	3	15	13	9	4	12	6	11	7	7
Task	Girls	2	1	0	0	0	0	0	1	0	0	2	0
	Boys	1	0	2	6	2	1	4	1	1	1	1	1
Environment	Girls	3	0	0	2	1	0	1	3	1	1	0	1
	Boys	3	4	3	6	6	2	4	1	2	3	2	2

*P^1 - Product, P^2 - Process, S - Spatial

Figure 28

Distribution of Unstable (effort and environment) versus Stable (ability and task) Attributions by Problem Type for Success and Failure (Table 7)

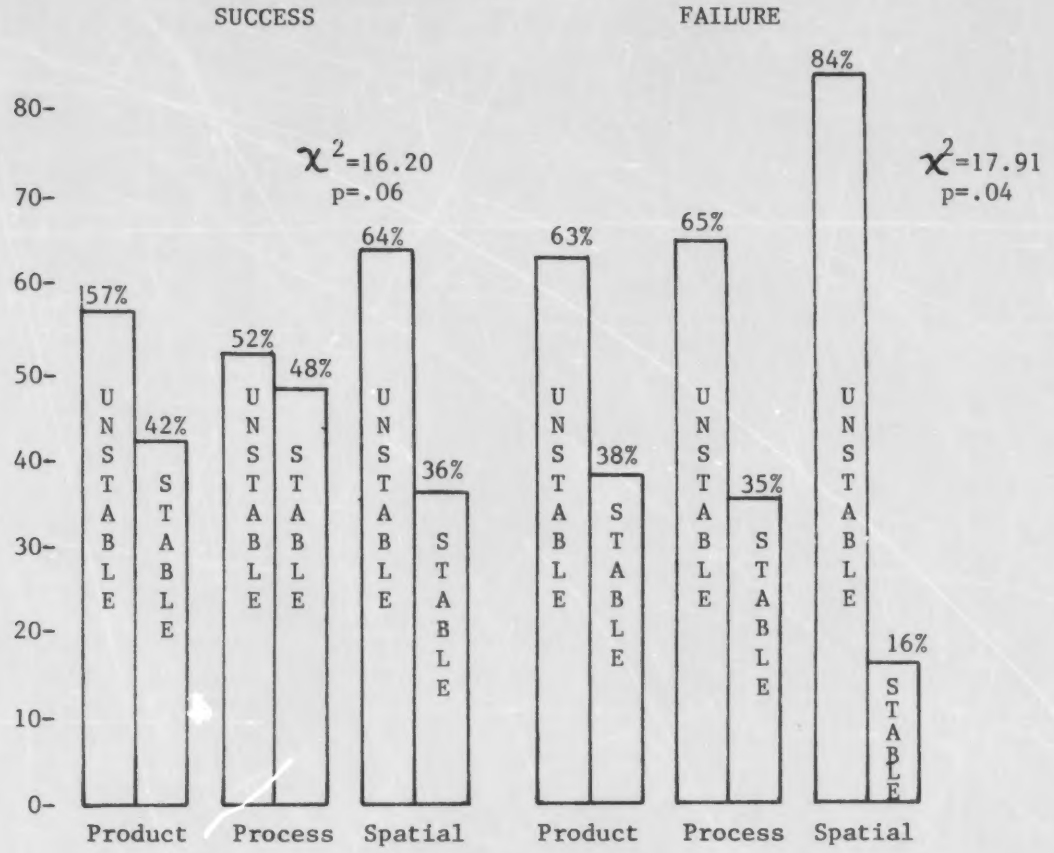


Figure 29

Success Attribution Patterns of High and Low Confidence Students (Table 7)

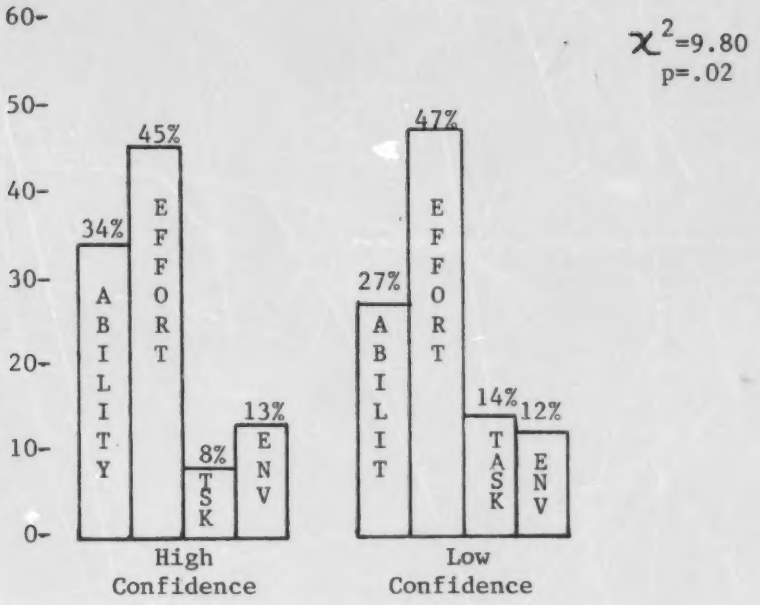
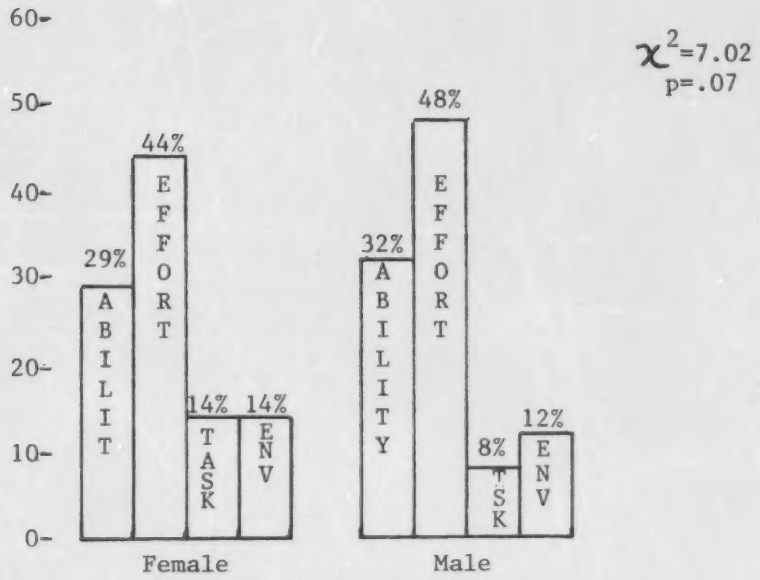


Figure 30

Success Attribution Patterns of Females and Males (Table 7)



external dimension, task and environment. We see that females make equal numbers of success attributions to task and environment, whereas males make more to environment. Note also that females' internal (ability plus effort) versus external (task plus environment) attribution ratio is lower than that for males; 73:28 for males versus 80:20 for males.

Finally, in Figure 31, we see one interesting four-way interaction when the analysis is restricted to success attributions with respect to product and process type problems alone. This effect seems to be due to the behavior of females with respect to process problems. This is the first time the attributions to ability and effort are seen to be equal. In all other ways of looking at the data, the effort attributions outnumber the ability attributions. This would seem to indicate that girls, at least, perceive process problems differently from product problems ... seemingly as a higher level cognitive task, one where ability is more a factor in the attribution of success than is effort. You have to be smart to do those kinds of problems.

Looking at the failure attributions overall, the analysis indicates that all the main effects other than confidence and sex are significant. In particular, students make many more failure attributions to boys than girls (almost three to one). Consistent with previous results, more students associate process, rather than product, problems with failure. 'Effort' is perceived by students as the main attribution for failure, as was the case for success, but no significant sex differences on failure attribution patterns are evident.

In a separate analysis of the attributions with respect to spatial problems, one effect, a confidence by attribution pattern interaction, is worth looking at. This result, seen in Figure 32 shows a higher rate of attributions to ability by high confidence students than by low confidence

Figure 31

Attribution Patterns for Product Problems of High and Low Confidence Females and Males (Table 7)

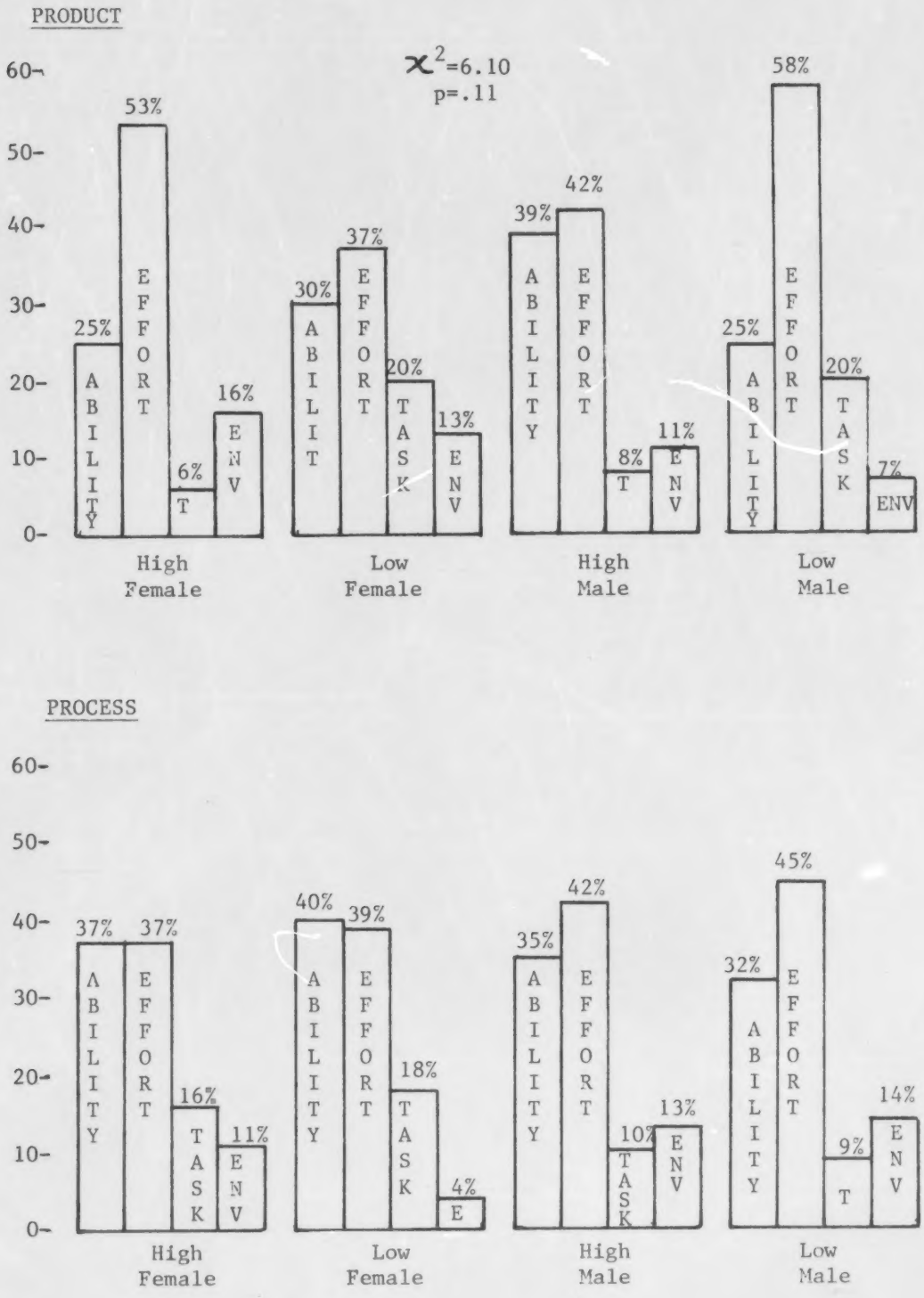
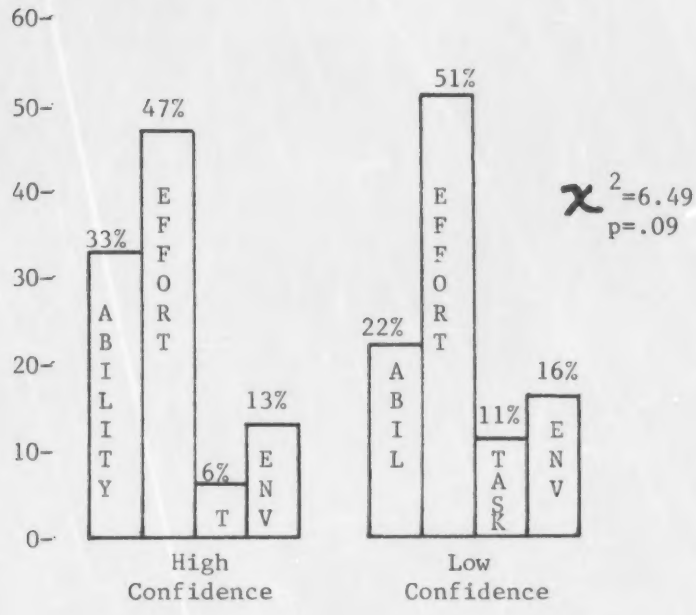


Figure 32

Patterns of Success Attributions Made by High and Low Confidence Students about Spatial Problems. (Table 7)



students. That is, high confidence students attribute more success in spatial problems to ability than do low confidence students. Low confidence students attribute more success to effort. This effect, described here for spatial problems, is consistent with that reported above for all problem types (See Figure 29), and is stronger here than when product and process problems are included as well.

Overall then, the attribution analysis indicates that all groups make more attributions to effort than to any of the other three categories. Further, high confidence students make a larger proportion of success attributions to ability than do low confidence students. There are no significant sex or confidence group differences with respect to failure attributions. The only sex differences in any of the attribution analyses are in the external dimension, task and environment; there, females make equal numbers of success attributions in each category, whereas males make fewer to the external dimension 'task and environment,' and more to the internal ones 'ability and effort.'

Discussion and Conclusions

The first results showed that the 1979 instrument generated many fewer responses in the 'failure' category than in the 'success' category. This was a result of a decision to emphasize positive aspects in the study. However, since the 'failure' category seemed to be generating some interesting results, the decision was made to revise the second year questions in order to create a more equal ratio of failure to success responses. These changes have been incorporated in the 1980 instrument, and it is expected that such changes will improve the validity of comparisons across the success/failure dimension of the study.

The expectation results are surprising. Here we see clearly that both males and females expect girls to do better in math than boys, and females

seem to believe so at an even higher rate than do males. Females and high confidence males perceive their teachers as having these same expectations as well. That is ... girls are better in math than boys. The interviews are liberally sprinkled with responses such as the following: From a couple of low confidence girls for example ...

"It seems like more girls are kind of a little smarter than boys. I mean, not saying that boys are stupid or nothing ... but it seems like girls are ... I don't know ... even when older, more teachers are women, so are smarter."

"Maybe girls are just smarter in math than boys."

Some low confidence boys remarked ...

"Girls are just smarter ... because they always get hundreds on their tests."

"Cause usually girls get them right quicker than boys."

And finally ... from a high confidence girl ...

"Girls are smarter than boys and boys are dumb but cute."

This result, the expectation that girls are better in math than boys, will be a particularly interesting one to observe longitudinally. Will this attitude change within the next two years? Certainly it is a surprising result in the light of the small number of women who end up in math-related fields, as well as the popular belief in math as a male domain. If math is not a male domain in the sixth grade, when does it become so? And, of course, <u>why</u> does it become so?

It isn't until we look at the 'feelings' responses that we come upon some findings that may be illuminating. Here, two pieces of information appear important.

First, high confidence females seem least positive about success ... and very hard on failure with respect to mathematics. We suggest this attitude may have an important influence on effectance motivation, a variable considered by many to be an important factor in encouraging people to continue the study of mathematics ... in fact, of any subject. Effectance motivation is the pleasure obtained in pursuit of an occupation or pastime for its own sake. These findings would suggest that the patterns of feelings such as those expressed here by high confidence females, might negatively affect effectance motivation in mathematics, and thus, be an important link between confidence and mathematics.

The second result that seems to offer some answers is the information that high confidence females are most uncomfortable, most negative, about coming to the teacher for help. Remarks made by high confidence girls illustrate this point ...

> "... if everything is really easy and nobody else had to go up there, you're the only one, somebody might think that she was really dumb."
>
> "... the teacher would think she's like stupid or something..."
>
> "... She might have been thinking well he might yell at me so maybe I shouldn't but then I'll never figure out how to do this problem. So I might as well."
>
> "... she would rather not ... because people would call her a dummy ... to come for help from the teacher."

The result is supported by findings in the classroom observation study where high confidence females were found to be involved in significantly fewer teacher-student interactions than any other group. Since high confidence girls would seem a most likely group to continue taking math courses, this group may be heavily handicapped by such feelings, particularly in a subject

such as mathematics where comfortable access to the teacher may be of paramount importance as the subject becomes more comples, and the necessity for clarification of concepts becomes more crucial.

Finally from the attribution analysis we see another result that may be important in providing answers to some of the questions raised in this study. Here we find that females seem to feel themselves less in control of their own successes than do males. This is seen in the lower internal to external attribution ratio for females than males (Figure 30). We also find that low confidence students make a greater proportion of effort to ability success attributions than do high confidence students. Since there are more low confidence girls than high confidence girls in the general population, this result indirectly supports the sex differences in success attribution patterns that have been hypothesized as an explanation of females' 'drop out' rate.* This is the theory that girls tend to attribute their successes more to effort and their failures to ability while boys do the opposite ... attribute more of their successes to ability and their failures to effort. Boys who make such attributions are less likely to perceive themselves blocked by failure (they can always work harder) whereas girls who tend more to perceive themselves blocked by ability, have no recourse but to drop out.

The failure attributions in these results are not significantly different for either confidence or sex, se we cannot say that the theory is completely supported ... at least not in our sixth grade data. However, we believe it is safe to say these data indicate that the females in our sample viewed success more outside their control. Perhaps next year, with more failure responses built into the instrument, we will see more complete support for the theory.

Needless to say, the findings reported here will be compared with results from the next two years of the study. We will be watching in particular to see whether the expectation patterns begin to change, whether the other interesting results are supported by subsequent data, and whether effects, not significant in the sixth grade data, begin to appear so later on.

The choice of sixth through eighth grades as those to be investigated in this study was made because of indications in the literature that many important attitude changes take place during this time. It will be of particular interest over the next two years, to see if this indeed proves to be the case.

Finally, I would like to close with the responses of a high confidence female to the question "How would this boy feel about getting the answer correct." To me, although she is speaking about a boy, this remark is extremely revealing about her <u>own</u> feelings toward success.

> "... He would feel good that he got the right answer. And he might feel sad though, that the other students don't like him as much because he gets the right answers more than they do."

V THE COGNITIVE COMPONENT

Because "mathematics is essentially cognitive in nature" (Weaver, 1971, p. 263) cognitive variables probably exert the most influence on the learning of mathematics. Of the many cognitive variables that influence mathematical learning, one that may help explain sex-related differences in mathematics achievement is spatial visualization--a particular subset of spatial skills. Spatial visualization involves mental imagery of objects and movement or change of the objects. In other words, objects or parts of objects, must be manipulated mentally. Even though the existence of many sex-related differences is being challenged, the evidence is still persuasive that, in the American culture, male superiority on tasks that require spatial visualization is evident beginning in adolescence (Fennema, 1975; Maccoby & Jacklin, 1974).

The relationship between mathematics and spatial visualization is logically evident and spatial representations are increasingly being included in the teaching of mathematics. However, empirical data does not always confirm that spatial skills are important in the learning of mathematics. Some investigators have definitely concluded that spatial skills and learning of mathematics are not related (Very, 1967; Fruchter, 1954).

Smith (1964) concluded . . . "the question whether mathematical ability is dependent on the visual factor (or factors) has not been definitely answered" (p. 68). Other investigators feel that data indicate a positive relationship (Guilford, Green & Christensen, 1951; French, 1951, 1955; Aiken, 1971). The Fennema-Sherman studies (1977) found that mathematics achievement and spatial visualization were related ($r = .5$) similarly for both females and males.

The concurrent development of sex-related differences in favor of males in both mathematics achievement and spatial visualization skills suggests the importance of this relationship. No significant sex-related differences in either mathematics achievement or spatial visualization skills have been consistently reported in subjects 7 to 8 years old. Sex-related differences in performance on spatial visualization tasks are noticed more often between upper elementary years and the last year of high school. In addition, the magnitude of the differences show a marked increase during this time span (Maccoby & Jacklin, 1974). Such sex-related differences in mathematics achievement as do exist also appear during this period (Fennema, 1974).

Several recent studies do indicate that spatial visualization skills are related differently to females' and males' learning of mathematics (Sherman, 1980). However, what this differential relationship is has not been explored. In addition, it is unknown how spatial visualization skills are utilized in mathematical thinking and problem-solving.

Another cognitive ability also important in mathematical thinking is verbal ability. Verbal ability as measured by language tests of various kinds (vocabulary, spelling, grammar, reading comprehension, reasoning) correlates highly with mathematics performance (Aiken, 1971; Fennema & Sherman, 1977). Some investigators have been concerned with mathematical

strategies involving both spatial and verbal skills. The idea of "different types of minds" was prevelant in the earliest days of psychology (Hadamard, 1954) and is currently seen in the study of differences in information processing in children (Kagan, Moss & Sigel, 1963).

It is in the relative use of spatial and verbal skills that a possible explanation of sex-related differences in mathematics may be found: 1) females, because of lesser development of spatial visualization skills have to rely more on verbal skills while males are more able to utilize either verbal or spatial skills? Some data supports this as an important hypothesis. Werdelin (1961) concluded that if one could attack a problem either verbally or spatially, one would be more apt to be able to solve it. His data showed that boys were superior on items which measured the ability to comprehend the organization of a visual figure and to reorganize it. Where items could be solved by verbal means and did not require that problems be translated into a mental figure, no sex-related differences were found.

It appears reasonable to believe that if males have developed more types of abilities than females or can use them more flexibly, then they should be able to attack mathematical problems in a variety of ways, score higher on mathematical tests, and thus, be encouraged to continue their study of mathematics. However, there is little research to verify this. Schonberger (1976) found that high ability females drew or modified diagrams for significantly more problems and did so correctly more often than did high or low ability males. In addition, spatial variables were more closely related to problem-solving by females than by males. It was Schonberger's hypothesis that this closer relationship was due to sex-related differences

in method of solving spatial visualization items. Males used more visual methods while females used more verbal methods. Some support for this result was found in the review of studies which included tests of figural reasoning as well as tests of spatial visualization and mathematical problem-solving (Flanagan, et al., 1964; Harris & Harris, 1973).

In summary, it appears that the only cognitive variable which may help in understanding why sex-related differences in mathematics exist is spatial visualization. The most promising area for investigation of this variable is its use together with verbal ability in mathematical thinking.

THE SPATIAL STUDY

The hypothesis under investigation is that the critical relationship between mathematics and spatial visualization is not direct, but quite indirect. It involves the translation of words and/or mathematical symbols into a form where spatial visualization skills can be utilized. The purpose of the Spatial Study is to provide information about spatial and verbal skills and their influence on the mathematical thinking of middle school pupils Mathematical Thinking is used in this study to describe the representations and procedures used by students to interpret and solve a variety of mathematical problems.

As a basis for developing the first Instrument to Measure Mathematical Thinking (IMMT-1) four questions were identified in terms of students who differed on specific cognitive skills.

Do females and males who differ on spatial visualization and verbal skills also differ:

1. in their verbalization of mathematical problems?
2. in their translation of mathematical problems into pictures and in their utilization of those pictures in the solutions of the problems?
3. in the method of problem solution with which they have the most success?
4. in the number of hints they need and in the number of correct answers they obtain when solving mathematical problems?

Providing answers to these questions in terms of sex differences, in terms of differences between the groups who differ on the cognitive skills, and in terms of the females and males within the cognitive skill groups could provide some information about the influence of spatial skills on sex-related differences in mathematics learning.

Sample Identification

To maximize the possibility of obtaining results, the sample for this study was selected using the following procedures. The sixth grade population (N = 676) of four middle schools (grades 6-8) in a midwest city was divided into thirds based on scores on Space Relations of the Differential Aptitude Test (Bennett, Seashore & Wesman, 1973) and scores on the Vocabulary Test of the Cognitives Abilities Test: Verbal Battery (Thorndike & Hagen, 1975). The students who scored in the upper third on Space Relations and in the lower third on Vocabulary Test were placed in the High Spatial/Low Verbal Sample; those who scored in the lower third on Space Relations and upper third on Vocabulary Test were placed in the Low Spatial/High Verbal Sample. In this manner two groups of students discrepant on Spatial/Verbal skills were identified. Each group consisted of 42 students divided equally among females and males from the sixth grade population. Table 8 gives the Means, Standard Deviations, and N's for each group on Space Relations and Vocabulary Test.

Instrument

The First Instrument to Measure Mathematical Thinking (IMMT-1) was composed of word problems and fraction problems. The problems represented mathematical questions determined as appropriate for 6th grade students through a series of pilot testings. The problem included originals developed by the staff as well as problems selected from various sources. The problems were routine in the sense that no "tricks" were needed to find the solutions. Five word problems and three fraction problems were identified for solution with pictorial representations. Five word problems were identified for analytic solution

Table 8
Means and Standard Deviations on Space Relations and Vocabulary Test for Discrepant Groups for Mathematical Thinking Sample

		Vocabulary Test	
		Low (N = 42)	High (N = 42)
Space Relations	Low		Verbal: $\bar{x}$ = 20.63 s.d.= 2.54 Range: 18-33 Spatial: $\bar{x}$ = 16.24 s.d.= 2.53 Range: 10-19
	High	Verbal: $\bar{x}$ = 13.12 s.d.= 2.25 Range: 5-15 Spatial: $\bar{x}$ = 31.91 s.d.= 4.63 Range: 10-42	

using only numerals. The analytic solution and the pictorial solution problems were similar in that the same procedures and mathematical operations were involved in finding the answers. For each problem a series of hints were designed. In the hint sequence, the first hint represented the minimal information, either pictorial or symbolic, that could be used as a starting point for a solution. The second hint included the first hint and also provided additional information from the problem. The final hint included the first and second hints and completely represented the problem. If labels were necessary, they were found in the last hint. Let me illustrate each type of problem, including the hints and briefly comment about how spatial visualization skills could

be utilized in finding a solution for the pictorial and fraction problems. Consider the following problem:

> A post 12 feet long is pounded into the bottom of a river near its bank. Two and a half feet of the post is below the bottom of the river and 1/2 foot is above the surface of the water. How deep is the river at that point?

Figure 31 Post Problem

For children of 11-12 years of age, this is a moderately difficult problem. You must add the lengths of two pieces of the post and then subtract that length from the total length, i.e., $2\frac{1}{2} + \frac{1}{2} = 3$ and $12 - 3 = 9$. Keeping track of the steps and sequencing them accurately is not easy. Consider the problem from a spatial visualization perspective. If one can visualize in one's mind what is involved, a picture like the one shown in Figure32 might be seen. The solution of the problem then becomes simpler. One has an image that enables you to move the pieces above and below the water together, then that length can be subtracted from the total length in order to get the correct answer. Once the problem can be visualized, spatial visualization skills can be utilized as a major aid in attaining a solution. The hints that are provided for this problem are in Figure33. The first gives the representation of a pole, a possible beginning point. The second represents the pole along with an indication of a region between two designated places, one near the top, the other near the bottom. The last hint lables the portions above and below the surface of the water and also gives the height of the pole. In the last hint the intent is to provide a pictorial representation of the complete problem.

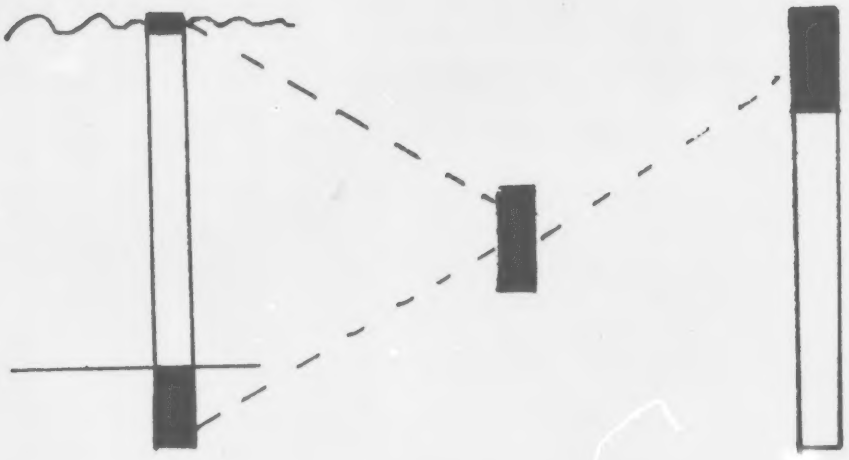


Figure 32

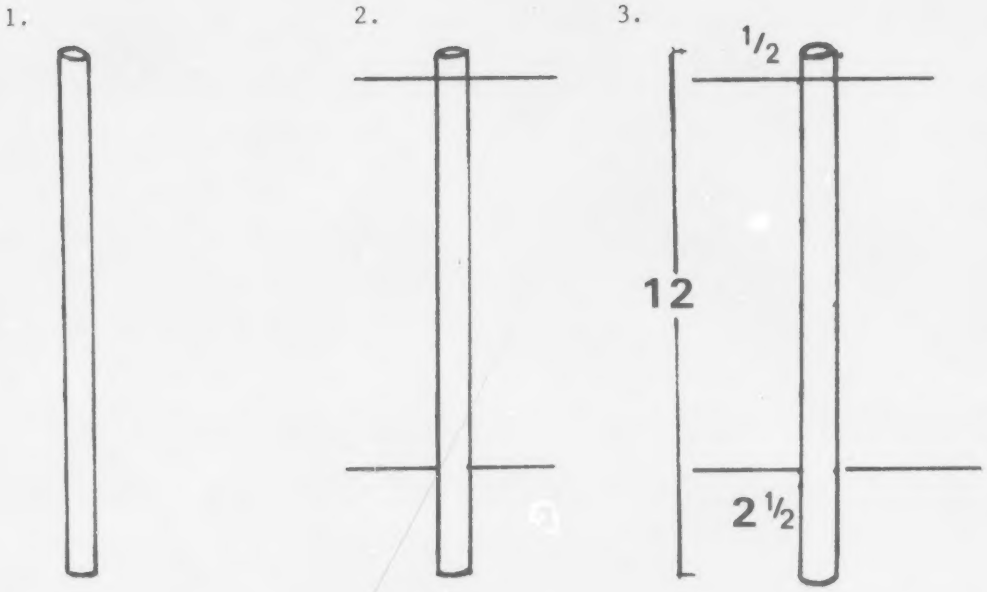


Figure 33

As another example, consider the fraction problem 2/3 - 1/2 =. While it can be solved totally with symbols, students of this age, because of their developmental level, often have trouble really understanding the symbolic process involved. Now look at the problem represented pictorially much as it could be visualized in the mind (Figure 33. Once it is visualized in the mind, spatial visualization skills can be utilized and the answer more easily found. The hints for the problem are given in Figure 34

As an example of the analytic word problems in IMMT-1 is given by:

> Gerri went to the sporting goods store and bought a $24.98 sleeping bag and a $2.39 canteen. Gerri gave the clerk $30.00. How much change did the clerk give back to Gerri?

Figure 636

Here the student needs to find the total cost of the purchase and then subtract that from the amount given to the clerk, $30.00. This problem is comparable to the post problem in that two quantities need to be added together and then subtracted from a third for the answer. The series of hints provided for this problem are given in Figure 37. The last hint represents the problem completely in a symbolic manner in that the student would obtain the correct answer by following the procedure given in the hint.

Figure 34

Figure 35

1. $24.98 $2.39

2. Cost of Bag $24.98
   Cost of Canteen 2.39

3. $ 24.98
   + 2.39 $30.00

Figure 37

Procedures

The students selected for the Mathematical Thinking Sample were asked to solve the problems on the IMMT-1 during individual interviews. There were three phases in the interview: Verbalization, Solution, and Explanation. Each phase was related to answering one of the major questions of the study. In the Verbalization phase, students were asked to read the problem silently and then to tell the interviewer what the problem was in their own words. During the Solution phase, students were asked to solve the problem using either a pictorial or an analytical procedure. When using the pictorial procedure, students were asked to draw a picture (called the translation picture) which would help them solve the verbal or fraction problems. In the analytical procedure, students were asked to use numbers to solve a verbal problem. If the students could not solve the problem, the hints were furnished as needed. During the Explanation phase, students were asked how they had used the picture to solve the problem (pictorial solution) or how they had solved the problem (analytical solution).

Categories

The interviews with the students were audio recorded and records were kept of the pictures drawn, the number of hints required, and the answers to the problems. The audio tapes were transcribed and the resulting protocols, pictures, and interview records were used to establish specific coding categories. Analyses were both quantitative and qualitative. The quantitative analyses focused on the categories and the qualitative analysis focused on what the students had drawn in their translation pictures. Table 9 gives the categories with the

solution procedures and problem types. For the Pictorial Solution

Table 9

IMMT-1 Procedures and Categories

Solution Procedures and Problem Types	Verbal Information	Picture Information	Picture Use	Mental Movement	Score
Pictorial					
Word Problems	X	X	X	X	X
Fractions	X	X	X	X	X
Analytical					
Word Problems	X			X	X

five categories were defined: Verbal Information, Picture Information, Picture Use, Mental Movement, and Score. The same categories were used for the Analytic Solutions except for Picture Information and Picture Use. Scores for each problem type were obtained by summing across all solutions of a specific type of problem. Table 10 identifies the components of the five categories. Some of these categories are self explanatory and others need some explanation. I'll briefly describe each.

Verbal Information. This category provides information regarding the verbalization of the problem: three specific areas for the pictorial and analytical problems and three somewhat similar areas for the fraction problems. For the pictorial and analytical problems the interest was in determining:

1) if the student gave all of the numerical as well as non-numerical information in his/her verbalization;
2) was that information properly placed; and
3) when stating the question was it given as presented in the problem or was there some translation, or was the question given incorrectly.

Table 10

Category Components by Problem Type

VERBAL INFORMATION	
Pictorial and Analytical	Fraction
Numerical Information	Numerical Information
Non-Numerical Information	
Problem Question	Problem Order
	Result of Operation

PICTURE INFORMATION (Translation Picture)	
Pictorial	Fraction
Numerical Information	Model
Non-Numerical Information	Operation/Denominator
Relations	Completeness
	Solution Form

PICTURE USE	
Pictorial	Fraction
Student's oral explanation of use of picture.	

MENTAL MOVEMENT	
Pictorial and Analytical	Fraction
Verbalization:	Verbalization:
Non-Numerical Data Transformed	
Questions Transformed	Question Transformed
Explanation:	Explanation:
Evidence from protocol	Evidence from protocol

SCORE	
Pictorial and Analytical	Fraction
Correctness of Answer	Correctness of Answer
Level of Hint Required	Level of Hint Required

For the fraction problems interest was in determining:

1) if the student gave all of the numerical data;
2) if an awareness of the result of the operation was indicated; and
3) if the information was stated in the order it was given in the problem.

Picture Information. This category provides information regarding how much the student represents pictorially on his/her translation of the problem into a pictorial representation. The information was analogous to that provided in the verbal information category for the pictorial problems:

1) the amount of numerical information provided,
2) the amount of non-numerical information indicated; and

also included was:

3) the indication of the relation stated in the problem; i.e. for the pole problem, the pole related to the top and bottom of the river.

Each of the relations coded in this category was a function of the individual problem. In this category, the fraction problems differed significantly from the pictorial problems. Four areas of interest were identified:

1) the model the students used in the pictorial representation;
2) the representation of the operation and common denominators;
3) the completenss of the picture with respect to including both fractions;
4) the form of the solution.

Picture Use. This category represents the student's oral explanation of his or her use of the picture in the solution. The evidence was provided by an overt expression of using the picture. An example of this category is provided in the following:

> "This is the pole that's being pounded into the river. Here is the um, let's see what part is this. This is the 2½ part and right here is the half foot, and this part is sticking up and this part is in, so I just figured out."

Mental Movement. This category identifies evidence of mental movement obtained from the verbal information supplied by the student as well as the oral explanation of the use of the picture in the solution. For the pictorial and analytical problems, the interest was in determining:

1) verbalization of some translation of the non-numerical data;
2) verbalization of some translation of the question;
3) some mental movement identified in the oral explanation.

For the fraction problems the interest was in determining:

1) verbalization of some translation of the question; and
2) some mental movement identified in the oral explanation.

Score. This category provides information regarding the correctness of the answer and the number of hints required for the solution.

In each of the categories, numerical scores were assigned from 0 to 4 with the highest representing complete information, total use of picture, and greatest evidence of mental movement. In SCORE, 4 = correct with no hints, 1 = correct with 3 hints, 0 = incorrect.

Analyses

To answer the questions, analyses of variances were performed using sex and mathematics thinking groups as sources of variance. Tables 11, 12 and 13, give the descriptive statistics, F-ratios, and p-values for the various groups. Significant differences were found between females and males on Pictorial Problem Score and Fraction Problem Scores. In each case, the males scored higher in those categories than did the females. High Spatial/Low Verbal students differed significantly from Low Spatial/High Verbal students in Picture Use on the fraction problems. The High Spatial/Low Verbal students made less use of the picture in their solutions to the fraction problems than did the Low Spatial/High Verbal students. The Low Spatial/High Verbal males had the highest score on Pictorial Problem Score

Table 11

Means, Standard Deviations, and F Scores for Analysis Categories by Sex

Categories	Female		Male		F	p
Pictorial Problems	$\bar{x}$	s.d.	$\bar{x}$	s.d.		
Verbal Information	36.10	6.23	37.05	5.94	.48	.49
Picture Information	25.32	8.31	26.20	8.12	.23	.63
Picture Use	8.62	1.37	8.85	1.50	.46	.50
Mental Movement	14.60	1.52	14.06	1.49	2.27	.14
Score	11.60	5.11	13.42	3.51	4.39	.04
Fraction Problems						
Verbal Information	6.83	1.91	6.33	3.14	.71	.40
Picture Information	11.27	5.14	12.39	6.58	.74	.39
Picture Use	3.82	1.40	4.12	1.87	.54	.47
Mental Movement	6.18	1.64	5.72	1.52	1.44	.23
Score	5.68	4.57	7.90	4.32	5.10	.03
Analytical Problems						
Verbal Information	38.70	5.66	38.77	4.54	.00	.95
Mental Movement	14.08	.98	13.90	.94	.69	.41
Score	12.49	5.24	14.18	4.15	2.54	.11

Table 12

Means, Standard Deviations, and F Scores for Analysis Categories by Mathematical Thinking Group

Categories	High Spatial/Low Verbal		Low Spatial/High Verbal		F	p
	$\bar{x}$	s.d.	$\bar{x}$	s.d.		
Pictorial Problems						
Verbal Information	35.54	6.39	37.56	5.63	2.20	.14
Picture Information	25.71	7.96	25.80	8.50	.002	.96
Picture Use	8.69	1.24	8.78	1.64	.06	.81
Mental Movement	14.28	1.52	14.38	1.54	.08	.78
Score	11.74	4.33	13.27	4.52	2.48	.12
Fraction Problems						
Verbal Information	6.59	2.68	6.58	2.52	.001	.98
Picture Information	12.46	6.42	11.20	5.33	.95	.33
Picture Use	3.39	1.40	4.63	1.68	10.95	.002
Mental Movement	6.12	1.34	5.78	1.78	.77	.38
Score	6.42	4.63	7.17	4.52	.56	.46
Analytical Problems						
Verbal Information	38.00	5.31	39.49	4.48	1.69	.20
Mental Movement	14.03	.96	13.95	.97	.13	.72
Score	12.78	4.77	13.87	4.81	1.04	.31
MT Score	31.25	11.08	34.18	11.92	1.28	.26

Table 13

Means, Standard Deviations, and F Scores for Analysis Categories by Mathematical Thinking Group and Sex

	High Spatial/Low Verbal				Low Spatial/High Verbal					
	Female		Male		Female		Male			
	$\bar{x}$	s.d.	$\bar{x}$	s.d.	$\bar{x}$	s.d.	$\bar{x}$	s.d.	F	p
Pictorial Problems										
Verbal Information	34.86	6.81	36.33	5.97	37.40	5.42	37.74	5.99	.92	.43
Picture Information	24.81	8.51	26.62	7.47	25.85	8.27	25.75	8.93	.17	.91
Picture Use	8.61	1.34	8.78	1.17	8.63	1.46	8.94	1.84	.18	.91
Mental Movement	14.47	1.54	14.06	1.52	14.75	1.53	14.06	1.51	.83	.48
Score	10.31	5.08	13.10	2.95	12.81	4.97	13.75	4.06	2.37	.08
Fraction Problems										
Verbal Information	6.30	2.08	6.90	3.23	7.35	1.60	5.80	3.04	1.39	.25
Picture Information	11.80	5.57	13.10	7.22	10.76	4.78	11.65	5.94	.55	.65
Picture Use	3.67	1.24	3.11	1.53	4.00	1.59	5.25	1.57	5.24	.001
Mental Movement	6.31	1.49	5.94	1.20	6.06	1.80	5.53	1.78	.75	.53
Score	5.45	4.66	7.33	4.51	5.91	4.59	8.50	4.15	1.94	.13
Analytical Problems										
Verbal Information	37.53	3.22	38.43	4.43	39.76	5.02	39.17	4.76	.69	.56
Mental Movement	14.10	.97	13.95	.97	14.06	1.03	13.85	.93	.26	.85
Score	11.75	5.18	13.76	4.23	13.19	5.33	14.67	4.13	1.27	.29

and Fraction Picture Use than did any of the other three groups of students.

Although statistically significant F scores were few, some possible areas of differences are suggested in Table 11, 12 and 13 Mental Movement for the Pictorial Problems and the Score on Analytic Problems might be categories which need to be considered in the last two years of the study in terms of possible sex differences. Verbal Information and Score on Pictorial Problems might be categories where differences, with respect to mathematical thinking groups, could appear at a later time. It appears that some differences are occuring although not dramatic ones.

Qualitative analysis

The purpose of the qualitative analysis was to describe the differences that can be identified between the students in the mathematical thinking sample in terms of the translation picture. Because some students needed hints to obtain the solutions to the problems this analysis considered only the translation pictures which were used in solving the problems. Although all translation pictures were interesting, not all clearly differentiated between the students in the sample. Three of the more promising differentiations were found in two pictorial problems and across the fraction problems. The first pictorial problem in which differences were identified was the post problem.

> A post 12 feet long is pounded into the bottom of a river near its bank. Two and a half feet of the post is below the bottom of the river and 1/2 foot is above the surface of the water. How deep is the river at that point?

Table 14 identified the translation picture with respect to mathematical thinking group and sex. Looking at the differences in the translation

picture in terms of sex differences finds 88% of the males and 73% of the females including a measure in their translation picture; 50% of the males and 50% of the females including two numerals; 17% of the males and 32% of the females including no numerals in their translation picture. Thirty-three percent of the males and 18% of the females included three numerals in their translation picture. The males seem to draw a more complete translation picture than do the females for the post problem. Looking at the data with respect to mathematical thinking groups finds 83% of the Hi/Lo students and 78% of the Lo/Hi students including a meausre in their translation picture; 48% of the Hi/Lo students and 52% of the Lo/Hi students include two numerals in their translation picture; and 26% of the Hi/Lo students and 22% of the Lo/Hi students including no numerals in their translation picture. Twenty-two percent of each group include three numerals in their translation picture. No real definitive pattern was identified here but the most identified translation picture, measured with 2 numerals, was drawn by 43% of the Hi/Lo students and 48% of the Lo/Hi students. Across the entire sample, 46% of the students drew that particular translation picture.

Table15 gives the characteristics of the translation picture for Problem 5: The bus problem.

> A bus has seats for 36 passengers. If one out of every four was empty, how many passengers were on the bus?

Sixty-five percent of the females and 48% of the males drew a representation where 1 of 4 items were crossed out during the solution of the problem. Only 12% of the females and 10% of the males drew a translation picture where no items were crossed out during the solution of the problem. Thirty-eight

Table 18

Characteristics of Translation Picture over All Correct Solutions of Problem 4 Post: Sex and Mathematical Thinking Group

(A Post with two distinct lines dividing it into three parts)

	Measure Included						No Measure Included					
	No Numerals		2 Numerals		3 Numerals		No Numerals		2 Numerals		3 Numerals	
	F	M	F	M	F	M	F	M	F	M	F	M
Hi/Lo	1	3 (4)	4	6 (10)	3	2 (5)	2	0 (2)	1	0 (1)	0	1 (1)
Lo/Hi	2	0 (2)	5	6 (11)	1	4 (5)	2	1 (3)	1	0 (1)	0	1 (1)
	3	3 (6)	9	12 (21)	4	6 (10)	4	1 (5)	2	0 (2)	0	2 (2)

Table 15

Characteristics of Translation Picture over All Correct Solutions of Problem # 5: Bus

	Crossed Out									
	Nothing		Column		Row		1 of 4		3 of 4	
Sex	F	M	F	M	F	M	F	M	F	M
Arrays	1	1	1	2	2	1	2	0	0	0
Rectangles	1	1	1	1	0	0	1	1	2	1
Groups	1	0	0	0	0	0	3	4	0	2
Seats							10	4	0	2
People							1	1		
	3	2	2	3	2	1	17	10	2	5

percent of the females and 29% of the males drew seats for their translation picture. The males represented the problem in a more abstract way; i.e. arrays, rectangles, groups, than did the females.

Tables 16 and 17 give the frequencies of types of translation picture in terms of mathematical thinking group and mathematical thinking group and sex respectively. Differences appear between the mathematical thinking groups with respect to the characteristics of the translation picture; 40% of the Hi/Lo students and 70% of the Lo/Hi students represent the crossing out of 1 of four items in their translation pictures; 20% of the Hi/Lo students and 4% of the Lo/Hi students show no crossing out in their translation pictures. Thirty-five percent the Hi/Lo students and 33% of the Lo/Hi students drew the seats in the translation picture. Although an apparent sex difference was identified in this characteristic, when combined across groups, this difference disappeared. Table 17 identified the Lo/Hi female students as those who most often crossed out one of four items in their translation pictures; 86% compared to 54% of the Lo/High males.

Table 18 gives the frequencies of types of translation pictures in terms of sex and mathematical thinking group for all fraction problems. The overriding model was an area model in which both fractions were represented. Differences with respect to sex appear in terms of how the students used that model. More females than males (56% to 29%) used the completed representation of both fractions, the common denominators, and the operation sign in their translation pictures. More females than males (68% to 43%) represented common denominators in their translation pictures. More males than females (57% to 32%) used no common denominator

Table 16
Characteristics of Translation Picture over
All Correct Solutions of Problem # 5: Bus
by Mathematical Thinking Group

	Nothing		Column		Crossed Out Row		1 of 4		3 of 4	
	Hi/Lo	Lo/Hi	Hi/Lo	Lo/Hi	Hi/Lo	Lo/HI	Hi/LO	Lo/Hi	Hi/Lo	Lo/Hi
Arrays	1	1	3	0	1	2	0	2	0	0
Rectangles	2	0	1	1	---		0	2	1	2
Groups	1	0	---				2	5	1	1
Seats	---		---		---		6	8	1	1
People	---		---				0	2	---	
	4	1	4	1	1	2	8	19	3	4

Table 17
Characteristics of Translation Picture over
All Correct Solution of Problem #5: Bus
by Sex and Mathematical Thinking Group

	Nothing				Column				Crossed Out Row				1 of 4				3 of 4			
	Hi/Lo		Lo/Hi		Hi/Lo		Lo/Hi		Hi/Lo		Lo/HI		Hi/Lo		Lo/HI		HI/Lo		Lo/Hi	
	F	M	F	M	F	M	F	M	F	M	F	M	F	M	F	M	F	M	F	M
Arrays	1	0	0	1	1	2	0	0	1	o	1	1	0	0	2	0	0	0	0	0
Rectangles	1	1	0	0	1	0	0	1	---				0	0	1	1	1	0	1	1
Groups	1	0	0	0	---								2	0	1	4	0	1	0	1
Seats	---												3	3	7	1	0	1	0	1
People	---												0	0	1	1	---			
	3	1	0	1	2	2	0	1	1	0	1	1	5	3	12	7	1	2	1	3

Table 18

Characteristics of Translation Picture over All Correct Solutions of the Fraction Problems by Sex and Mathematical Thinking Group (Model: Area with both fractions represented)

	No Common Denominator					Common Denominator				
	Operation sign			No Operation sign		Operation sign			No Operation sign	
	F	M		F	M	F	M		F	M
Hi/Lo	4	3	(7)	1	9 (10)	11	6	(17)	5	3 (8)
Lo/Hi	5	11	(16)	7	2 (9)	19	7	(26)	1	3 (4)
	9	14	(23)	8	11 (19)	30	13	(43)	6	6 (12)

in their translation pictures. The operation sign was included by 74% of the females and 61% of the males.

Looking at the information with respect to mathematical thinking group finds that 40% of the Hi/Lo students and 47% of the Lo/Hi students used the complete representation in their translation pictures. Common denominators were found in 60% of the Hi/Lo students and 55% of the Lo/Hi students translation pictures; 57% of the Hi/Lo students and 76% of the Lo/Hi students included the operation sign. The Lo/Hi students made use of the complete representation more than did the Hi/Lo students.

Conclusions

The information obtained during the first year of the Spatial study can be described from two views: the first representing a global description of a newly designed instrument using a different type of a technique in terms of identifying the influence of spatial/verbal skills on mathematical thinking. It does seem that the instrument has merit in terms of identifying how students translate verbal problems into pictures and does provide some information on how the picture is used in the solution. What has not happened have been statistically significant results with respect to sex differences or differences with respect to mathematical thinking groups. The coding of the instrument into various pieces and then the combining of those pieces into the analyses categories lead to this conclusion. Although the translation pictures have been described and some differences noted, further investigation on how the students arrived at those completed pictures might identify differences with respect to translation of word problems and fraction problems. The fraction problems gives the most information with respect to differences and that information has been used in redesigning the instrument for the

respect to translation of word problems and fraction problems. The fraction problems gave the most information with respect to differences and that result has been used in redesigning the instrument for the second year of the study.

The second view regarding the information obtained during the first year of the study is that of identifying questions regarding the instrument and also the importance of the contribution of spatial visualization and verbal skills to mathematical thinking. The problems in the instrument were coded in terms of categories that identified what the student was doing during the solution relative to the pictorial representation and not the specifics of how the problem was solved. Because the results only identified subtle differences, it might be that for this age group the influence of spatial visualization and verbal skill on mathematical thinking is minimal.

Notes

1. Fennema, E. and Carpenter, T. What the Second National Assessment of Educational Progress has to say about sex-related differences in mathematics. (In Progress)

References

Aiken, L.R Intellective variables and mathematics achievement: Directions for research. Journal of School Psychology, 1971, 9, 201-209.

Aiken, L.R. Ability and creativity in math. Review of Educational Research, 1973, 43, 405-432.

Bar-Tal, D. Attributional analysis of achievement related behavior. Review of Educational Research, 1978, 48(2) 259-271.

Bar-Tal, D and Frieze, J.H. Achievement motivation for males and females as a determinant of attributions for success and failure. Sex Roles, 1977, 3, 301-313.

Bennett, G.K., Seashore, H.G., & Wesman, A.G. Differential aptitude tests: Administrator's handbook. New York: The Psychological Corporation, 1973.

Brophy, J.E., & Good, T.L. Brophy-Good System (Teacher-Child Dyadic Interaction). In A. Simon and E. Boyer (Eds.), Mirrors for behavior: An anthology of observation instruments continued, 1970 supplement. Volume A. Philadelphia: Research for Better Schools, Inc., 1970.

Brophy, J.E., & Good, T.L. Teacher-student relationships: Causes and consequences. New York: Holt, Rinehart and Winston, 1974.

Deaux, K. Sex: A perspective on the attribution process. In Harvey, J., Ickes, W. and Kidd, R. (eds.) New Directions in Attribution Research (Vol. I) Hillsdale, New Jersey: Lawrence Earlbaum, Associates, 1976.

Dweck, C.S., & Bush, E.S. Sex differences in learned helplessness: I. Differential debilitation with peer and adult evaluators. Developmental Psychology, 1976, 12, 147-156.

Dweck, C.S., Davidson, W., Nelson, S., & Enna, B. Sex differences in learned helplessness: II. The contingencies of evaluative feedback in the classroom and III. An experimental analysis. Developmental Psychology, 1978, 14, 268-276.

Everitt, B.S. The analysis of contingency tables. New York: John Wiley and Sons, Inc., 1977.

French, J.W. The West Point tryout of the guidance battery. (Research Bulletins 41-12 and 55-6) Princeton, Educational Testing Service, 1951 and 1955.

Fennema, E. Mathematics learning and the sexes: A review. Journal for Research in Mathematics Education, 1974, 5, 126-129.

Fennema, E. Influences of selected cognitive, affective and educational variables on sex-related differences in mathematics learning and studying. In NIE Papers in Education and Work: No. 8. Women and Mathematics, 1977.

Fennema, E. Implications of research on sex-related differences in mathematics. In Fennema, E. (ed). Implications of research for the Curriculum of the 80's. ASCD (In Progress)

Fennema, E.,&Sherman, J. Sex-related differences in mathematics achievement, spatial visualization and affective factors. American Educational Research Journal, 1977, 14(1), 51-71.

Fennema, E., & Sherman, J.A. Sex-related differences in mathematics achievement and related factors: A further study. Journal for Research in Mathematics Education, 1978, 9(3), 189-203.

Flanagan, J.C. Changes in school levels of achievement: PROJECT TALENT ten and fifteen year retests. Educational Researcher, 1976, 5(8), 9-12.

Fruchter, B. Measurement of spatial abilities: History and background. Educational and Psychological Measurement, 1965, 25, 387-395.

Good, T.L., Sikes, J.N., & Brophy, J.E. Effects of teacher sex and student sex on classroom interaction. Journal of Educational Psychology, 1973, 65, 74-87.

Guilford, J.P., Green, R.F., & Christensen, P.R. A factor analytic study of reasoning abilities, II. Administration of test and analysis of results. Report No. 3, 1951. University of Southern California Psychology Laboratory.

Haberman, S.J. Log-linear fit for contingency tables, algorithm. Applied Statistics, 1972, 21, 218-224

Hadamard, J.S. Essay on psychology of invention in the mathematics field. New York: Dover Publications, 1954.

Harris, M.L., & Harris, C.W. A structure of concept attainment abilities. Wisconsin Monograph Series. Madison, Wisconsin, Research and Development Center for Cognitive Learning, University of Wisconsin, 1973.

Kagan, J., Moss, H.H., & Sigel, I.W. Psychological significance of styles of conceptualization. In J.C. Wright & J. Kagan (eds). Basic cognitive processes in children. Monograph for Sociological Research in Child Development, 1963, 28(2), 73-112.

Maccoby, E.E., & Jacklin, C.N. The Psychology of Sex Differences. Stanford: Stanford University Press, 1974.

Mullis, I.V.S. Educational achievement and sex discrimination. Denver, CO. National Assessment of Educational Progress, 1975.

Nelson, M.H. Mathematical ability, inability and disability. Unpublished masters thesis, Massachusetts Institute of Technology, 1977.

Schonberger, A.K. The interrelationship of sex, visual spatial abilities and mathematical problem solving ability in grade seven (Doctoral Dissertation) University of Wisconsin-Madison, 1976.

Sherman, J. Effects of biological factors on sex-related differences in mathematics achievement. In NIE Papers in Education and Work: No. 8, Women and Mathematics, 1977.

Smith, I.M. Spatial Ability. Knapp. San Diego, 1964.

Thorndike, R.L., & Hagen, E. Cognitive Abilities Test, Form 1 (Levels A-H). Boston: Houghton Mifflin, 1971.

Very, P.S. Differential factor structures in mathematical ability. Genetic Psychology Monographs, 1967, 75, 169-207.

Weaver, J.F. Seductive shibboleths. The Arithmetic Teacher, 18, 263-264.

Weiner, B. Achievement motivation and attribution theory. Morristown, N.J.: General Learning Press, 1974.

Werdelin, I. Geometrical ability and the space factor in boys and girls. Lund, Sweden: C.W.K. Gleerup, 1961.

Wolleat, P., Pedro, J.D., Becker, A., & Fennema, E. Sex differences in high school students' causal attributions of performance in mathematics. Journal for Research in Mathematics Education, In Press.